

## THERMAL DESIGN IN PERMAFROST SOILS

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Other papers presented at the Third Canadian Conference on Permafrost have clearly outlined the extent of permafrost and a great many of the problems associated with operations in permafrost regions. It has also been clearly shown that the permafrost environment is one in which great forces are at work and that these forces are in delicate balance. Small man-made disturbances can then be expected to trigger great undesirable changes unless care is exercised in maintaining the balance.

One method of maintaining the appropriate balance is to design facilities with the thermal regime predetermined and with the soil to be retained in the frozen state or purposefully melted at a predetermined rate and to a predetermined extent.

The object of this paper is to discuss the methods of calculating the thermal regime, and a set of examples are used to illustrate some design processes.

### THE NATURAL THERMAL REGIME

It has been shown by R. J. E. Brown that the seasonal surface temperature variation causes an annual temperature oscillation below the surface which decreases in amplitude with depth. The envelope surrounding all of the temperatures occurring through a year is called a "trumpet curve" and is schematically shown in Figure 1 with the geothermal gradient superimposed. General features on Figure 1 are interesting.

- The mean annual soil surface temperature is the extrapolation of the geothermal gradient to the ground surface.
- The surface temperature fluctuation can generally be assumed to fit the equation  $T_s = \text{MASST} + A \sin(\omega t - \alpha)$

where

$T_s$  = Surface Temperature

MASST = Mean Annual Surface Temperature

A = Amplitude of Seasonal Variation of Surface Temperature.

- The shape of the "trumpet curve" is shown for a homogeneous soil and one for which the effect of the phase change of water is insignificant.

The mathematical solution to the temperatures at various depths is well known and available from numerous sources (Schneider, 1957).

One solution is:

$$T = \text{MASST} + A [e^{-\sqrt{\omega/2\alpha x}} \cos(\omega t - \sqrt{\omega/2\alpha x})] \quad (1)$$

where

$$\begin{aligned} T_s &= \text{MASST} + A \cos \omega t \\ \alpha &= \text{Thermal Diffusivity} \\ x &= \text{Depth} \\ \omega t &= \text{Time} \\ \omega/2\pi &= \text{Frequency of Temperature Oscillation (1 year)} \end{aligned}$$

The depth,  $D$ , at which the seasonal variation becomes smaller than can be shown on the graph is dependent on both  $A$  and  $\alpha$ .

This description of the natural thermal regime is oversimplified in three respects. The first is the assumption that the soil is homogeneous and is not a layered system. The second is the assumption that the soil thermal properties are the same in the thawed and frozen states, and that the phase change of soil moisture is insignificant. The third is the assumption that the annual soil surface temperature variation is sinusoidal and the same from year to year. None of these assumptions is accurate, of course, and methods of incorporating these features in calculations will be shown in a later section. Many practical applications, however, do not require consideration of these complicating elements.

It will be noted that two interesting measurements can be made in the field by utilization of this relatively simple description of the thermal regime.

- A bottom hole temperature taken at a depth below  $D$  can be used to calculate MASST by correcting for the geothermal gradient from  $D$  to the ground surface.
- The effective thermal diffusivity,  $\alpha$ , can be determined by measuring the surface temperature amplitude  $A$  and several amplitudes,  $a$ , below the surface then solving for  $\alpha$  in equation 1 at each depth. This measurement will yield erroneous results if the soil is significantly heterogeneous.

The occurrence of permafrost is schematically shown in Figure 2 where permafrost is that soil which is never warmer than  $32^\circ\text{F}$  during the year. The active layer at the ground surface is seasonally thawed and frozen, and the soil below the permafrost layer has very little seasonal variation and is above freezing.

#### MAN-MADE THERMAL CHANGES

Virtually everything that man does to the ground surface in permafrost regions has a tendency to increase the amplitude of seasonal

variation,  $A$ , and the mean annual soil surface temperature, MASST. The effect of increasing  $A$  on permafrost thawing is shown in Figure 3 where the geothermal gradient is sketched much more steeply than actually occurs and shows that increasing the surface amplitude causes thawing near the surface but has no effect on the maximum depth of the frozen material.

Roads are a good example of a thermal disturbance which increases  $A$ . Snow removal during the winter causes a decrease in winter temperature, and destruction of plants and decreasing evaporation causes an increase in summer temperatures.

Increasing MASST causes thawing on both the upper and lower permafrost boundaries as shown in Figure 4. Here again road construction is a good example of increasing the MAAST; however, it should be noted that a road does not cover enough ground surface to cause a significant disturbance to the lower boundary in regions of thick permafrost.

The combination of both an increase in  $A$  and MASST is shown in Figure 5 with thawing on both the upper and lower permafrost boundaries.

Much has been written about the serious consequences arising from thawing ice rich permafrost. These undesirable situations will not be discussed here, but one of the more important facets of construction on permafrost soils is the calculation of the settlement which results from melting. The ice content of soils is highly variable (from 0% to 100% by volume), and one cannot generalize by stating that permafrost must always be retained in the frozen state or vice versa. Each case must be individually evaluated, and acceptable solutions will be greatly variable. A road location is sometimes possible which bypasses almost all ice rich soils and passes over soils which will experience negligible settlement when thawed. The point to be made here is that good design requires both knowledge of the soils and their temperatures after construction. Such a design is sometimes trivial under certain situations and very difficult in others.

### STEADY STATE SOLUTIONS

Many design problems can be properly solved by using relatively simple steady state heat flow calculations. Examples include the calculation of thaw basin geometry under buildings, around pipelines, and under roads and airfields. In these cases of thaw basin geometry it is not generally necessary to take seasonal temperature variation into consideration.

The great advantage to steady state analysis is that superposition can be applied, and this allows the problem to be broken into small and manageable sub-problems which can then be added together for a complete solution.

### The Two-Lane Road Case

A very useful, analytical method using superposition by an easily understood graphical method has been published (Brown, 1963). The basic element of this technique is shown in Figure 6 where the heat flow lines are segments of circles and the isotherms are equally spaced radial lines. Two regions are shown of different mean annual soil surface temperatures such as a beach line or the edge of a very large building.

Two solutions of this type can be superimposed as shown in Figures 7 and 8 where the disturbance caused by the heated region is added to the original uniform soil temperature. The isotherms are then constructed by contouring the many individual points of equal temperature at intersections of radial lines. Although this sketch shows a radial line for each 0.2 increment of disturbance for clarity, a much smaller increment can be easily constructed. The isotherms shown are true circular segments.

Two disturbances can be added as shown in Figure 9, and in addition, the geothermal gradient can be added as shown in Figure 10 in which the basin of thawed soil is indicated. It is again emphasized that this type of solution assumes a homogeneous soil, and that a layered system of soils with variable thermal conductivity will have a different thermal regime. The layered case can be solved by superposition, but not directly by this method.

The thaw basin shown is the equilibrium condition that will be achieved over a period of time after the disturbance was made. The theoretical time to equilibrium is infinite, but for practical purposes the time to equilibrium is short compared to the lifetime of the installation in most cases.

### The Heated Pipe Case

Another steady state solution is the case of a heated pipeline such as one carrying warm crude oil. This case is relatively simple because a solution has been analytically solved in closed form (Lachenbruch, 1958).

The equation for heat lost from the pipe to the atmosphere in steady state is as follows with the nomenclature shown in Figure 11.

$$Q = \frac{4\pi kL v_o}{\ln \{ [8(N/D)^2 - 1] + 4(N/D) \sqrt{4(N/D)^2 - 1} \}}$$

for  $N/D \gg 1$

$$Q = \frac{2\pi kL v_o}{\ln 4(N/D)}$$

where

$k$  = Thermal Conductivity of the Soil

$L$  = Length of Pipeline being Considered (1 foot)

$v_o = t_o - t_s$

The geometry of the isotherms is determined from the following equations, and a representative solution is shown in Figure 12.

$$y = N(1+C)/(1-C)$$

$$r = 2N\sqrt{C}/(1-C)$$

$$C = e^{(4\pi kL/Q)v}$$

$$v = t - t_o$$

The heat loss from the fluid in the pipe will cause a temperature loss in the fluid and this temperature change can be calculated stepwise over the length of the line. A sketch of such results is shown in Figure 13.

### The Heated Building Case

Problems which require a three-dimensional solution are considerably more difficult to analyze than the previous two-dimensional case. Fortunately, an excellent analytical procedure with many applications has been developed (Lachenbruch, 1958). This procedure allows a rather complete determination of the thermal regime under disturbed ground surface areas of any shape and is applicable to problems associated with buildings, roads, airfields, lakes, and shorelines among others.

The example used is a circular warm oil storage tank 200 feet in diameter, and two features of the soil temperature will be determined for a point 100 feet beneath the centre of the tank. The first feature is the equilibrium temperature that will be reached after a long time, and the second feature is the temperature at the end of one year after construction.

No attempt will be made in this example to reproduce Lachenbruch's charts and graphs with accuracy because their physical size is too large. It is highly recommended that the reader obtain an original copy of the Bulletin.

The analysis uses a chart which represents the ground surface as shown in Figure 14. Each small "rectangular" space ( $\Omega$ ) represents 1/1000 of the thermal disturbance that would be felt at a point directly under the centre of the chart at a depth,  $z$ . In other words, if a building had a floor temperature  $10^\circ\text{F}$  warmer than the average soil temperature, and if the building drawn to scale covered 500 "rectangles", then the disturbance at depth  $z$  would be  $5^\circ\text{F}$  warmer or  $(500/1000)(10^\circ\text{F}) = 5^\circ\text{F}$ . The scale of the drawing of the building area is dictated by the depth being considered. If a depth of 20 feet is to be studied, then the length "z" shown in Figure 14 equals 20 feet and the building floor must be drawn to this scale. The tank is shown on the drawing as a solid line in the location to determine the temperature under the centre at a depth of 100 feet. If the soil temperature were to be determined under the edge of the tank at a depth of 200 feet, the drawing would be located as shown by the dotted line.

The temperatures assumed for this example are as follows:

Undisturbed Region

Soil Surface Temperature

$$T_s = 10^\circ\text{F} + 20 \sin(\omega t - \alpha)^\circ\text{F}$$

Mean Annual Soil Surface Temperature

$$\text{MASST} = 10^\circ\text{F}$$

Tank Bottom Temperature

Bottom Surface Temperature

$$T_d = 40^\circ\text{F} + 10 \sin(\omega t - \alpha)^\circ\text{F}$$

Mean Annual Tank Bottom Temperature

$$\text{MATBT} = 40^\circ\text{F}$$

The Geothermal Gradient =  $2^\circ\text{F}/100$  feet.

The steady state disturbance is then

$$B = \text{MATBT} - \text{MASST} = 40 - 10 = 30^\circ\text{F}$$

The first feature of the temperature field to be calculated is the steady state equilibrium temperature beneath the centre of the tank at a depth of 100 feet. All that is needed is to count the number of "rectangles" covered by the tank on the chart in Figure 14, and this is shown in Table 1, column 1.

The second feature of the temperature field to be calculated is the temperature at the end of one year after construction. This calculation requires the use of another Lachenbruch chart shown roughly in Figure 15 and from which is determined a time delay weighting factor,  $\phi$ .

There is a unique value of  $\phi$  for each annular zone of Figure 14 because of the greater heat path distance with increasing zone radius. The results of these calculations are shown in Table 2 for an assumed soil thermal diffusivity,  $\alpha$ , of  $0.012 \text{ cm}^2/\text{sec}$  for ice laden silt. For the one year time period,  $\alpha t = 3.8 \times 10^5 \text{ cm}^2$ .

Although a circular tank was used for this example, many other applications are obvious.

### Numerical Solutions

There are several numerical techniques for solving steady state problems; however, only relaxation will be discussed here because of difficulties with other techniques in handling problems of heterogeneous soils and the change in soil properties from the frozen to unfrozen state. Numerical techniques are powerful tools in solving difficult problems, but have the disadvantage of being expensive. They also do not lend themselves well to general solutions.

The two-dimensional case will be used as an example and it can be seen that the three dimensional case is a simple extension.

The heat flow in Figure 16 is assumed to be either horizontal or vertical from square to square and the centre of each square is representative of the temperature for the entire square. The example shown in Figure 16 is almost trivial but illustrates the technique.

With steady state the summation of heat flow from surrounding squares into square number 1 must equal zero. The equations that follow are written for heat flow from centre to centre of volumes of material with unit thickness for unit time.

$$Q = \frac{kA\Delta T}{L}$$

$k$  = Thermal Conductivity

$A$  = Area

$\Delta T$  = Temperature Difference

$L$  = Flow Path Length

$$Q_{2-1} = \frac{(k)(\delta)(1)(\delta)(T_2 - T_1)}{\delta}$$

$$Q_{3-1} = (k)(\delta)(T_3 - T_1)$$

$$Q_{4-1} = (k)(\delta)(T_4 - T_1)$$

$$Q_{5-1} = (k)(\delta)(T_5 - T_1)$$

$$Q_{2-1} + Q_{3-1} + Q_{4-1} + Q_{5-1} = 0$$

$$(T_2 - T_1) + (T_3 - T_1) + (T_4 - T_1) + (T_5 - T_1) = 0$$

$$T_1 = \frac{T_2 + T_3 + T_4 + T_5}{4} \quad (2)$$

If the temperature array was estimated and equation 2 was applied to a set of 5 nodes, any error in the estimate would be evident. Using location "a" in Figure 17 for example, equation 2 would be as follows if the interior temperatures had been estimated to be zero:

$$0 = \frac{10 + 0 + 0 + 0}{4} = 2.5$$

The estimate for "a" to be zero is in error and the value of 2.5 is used instead. Now looking at location b:

$$0 = \frac{10 + 0 + 0 + 2.5}{4} = 3.1$$

This process can be repeated until the errors become insignificant, and these relaxation results are shown in Figure 17.

The relaxation method can be extended to include complicated systems such as shown in Figure 10 and also can be used to solve layered soils with the variable thermal conductivity between the frozen and unfrozen soil by solving for the appropriate relaxation equation in lieu of equation 2.

### TRANSIENT HEAT FLOW

Many practical problems of design on permafrost are involved with soils near the upper surface where the seasonal disturbance is great. Roads and airfields are good examples. It was pointed out earlier that road construction increases the mean annual soil surface temperature and the variation of seasonal soil surface temperature both of which increase the depth of seasonal thaw and can cause permafrost thawing. The solution to this problem is to apply enough insulation beneath the road surface to prevent permafrost degradation. The insulation can be almost any material, but soils are generally the cheapest and most readily available. The problem is then to select the fill thickness which will provide an adequate insulating blanket.

#### Two-Layer Case

Lachenbruch (1959) has again come to the rescue with a very simple method of solving the problem. In Figure 18 the original condition is shown with solid lines and the top of permafrost is indicated. The condition which would occur from road construction where the fill

thickness is adjusted to make the new trumpet curve through the fill match the original one at point M is also shown as dotted lines. If the match can be made at M, no permafrost will be melted.

Figure 19 shows the Lachenbruch solution in graphical form and two examples are calculated in Table 3. It is very important to notice that the soil upon which the road is constructed has great bearing on the fill requirement.

Lachenbruch's paper also discusses a three-layer solution which allows calculation using thin layers of man-made insulation.

### Numerical Solutions

Numerical solutions to transient heat flow problems show considerable promise in solving complicated problems of layered systems. A description of these techniques which are applicable to the more practical problems with permafrost is lengthy and beyond the scope of this paper. There are many publications which develop techniques for handling homogeneous materials one of which is included for reference (Schneider, 1957). A substantial extension to the iterative technique is being made by the University of Alaska which takes into account many layers variable thermal properties between the frozen and unfrozen state, heat of fusion, and changes in surface conditions such as road construction. An example of the type of results which can be achieved is shown in Figure 20.

### CONCLUSIONS

The objective here is to show that many problems associated with construction in permafrost regions can be solved by predicting the thermal regime quantitatively and designing accordingly. There are many methods of attacking such problems from a heat transfer point of view, and some of these methods have been discussed.

### REFERENCES

1. Brown, W.G. "Graphical Determination of Temperature Under Heated or Cooled Areas on the Ground Surface". Div. of Bldg. Res., Nat. Res. Council Tech. Paper No. 163, 1963.
2. Eckert and Drake. Heat and Mass Transfer. McGraw-Hill, 1959. 530 pp.
3. Lachenbruch, A.H. "Three Dimensional Heat Conduction in Permafrost Beneath Heated Buildings", U. S. Geological Survey Bull. 1052-B, 1958.

4. Lachenbruch, A.H. "Periodic Heat Flow in a Stratified Medium with Application to Permafrost Problems", U. S. Geological Survey Bull. 1083-A, 1959.
5. Schneider. Conduction Heat Transfer, 1957, 395 pp.

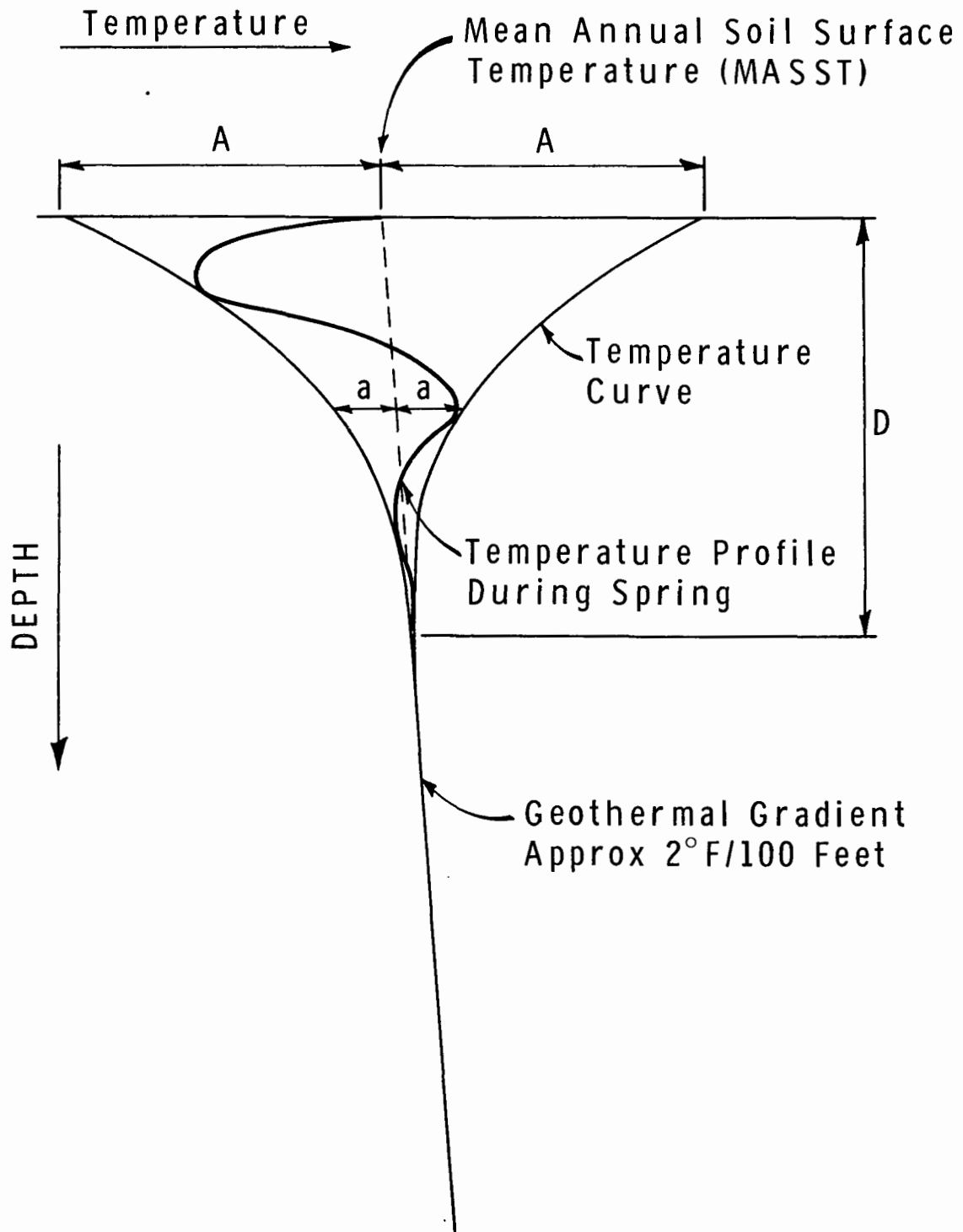


FIGURE 1  
NATURAL THERMAL REGIME

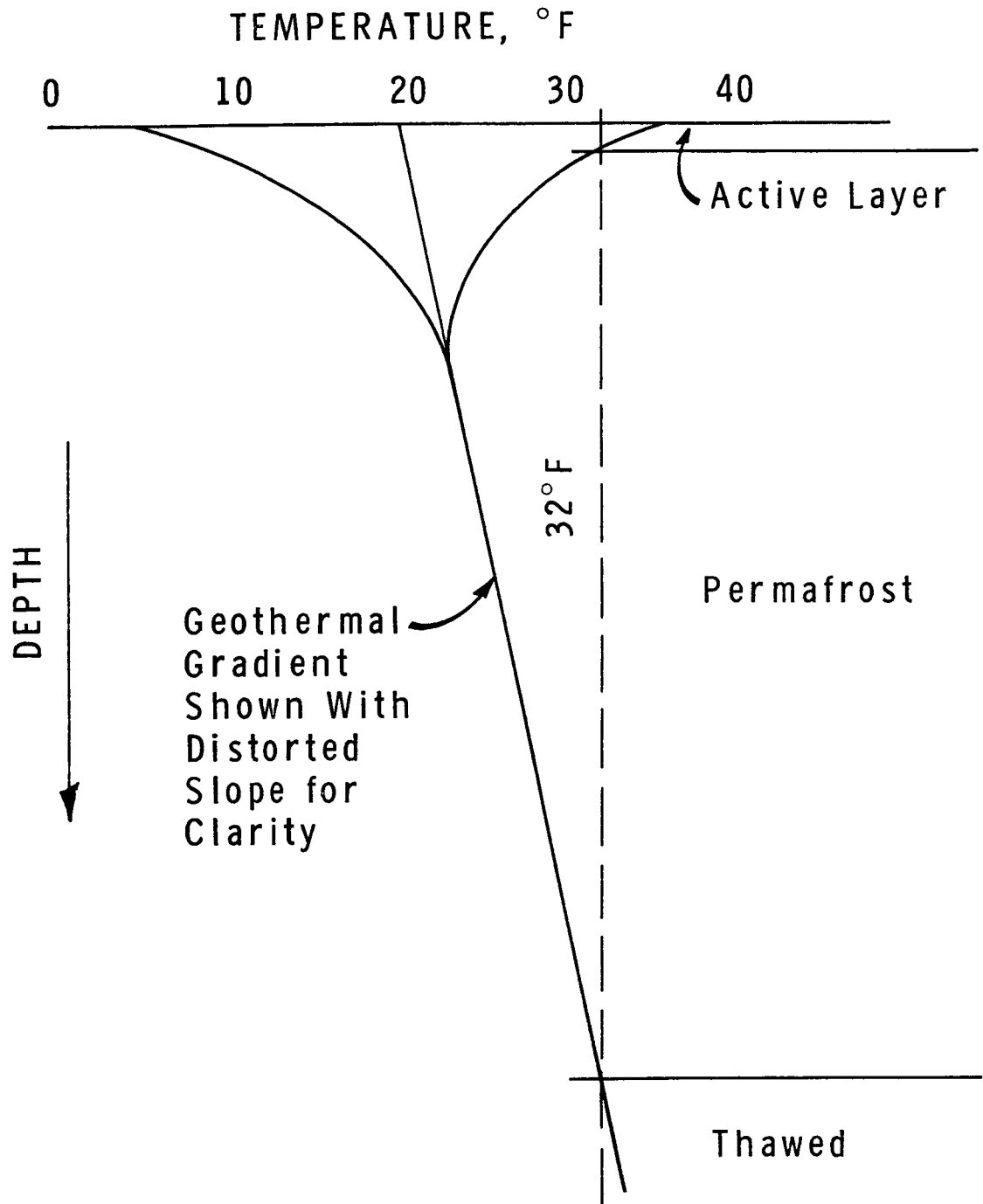


FIGURE 2 PERMAFROST OCCURRENCE

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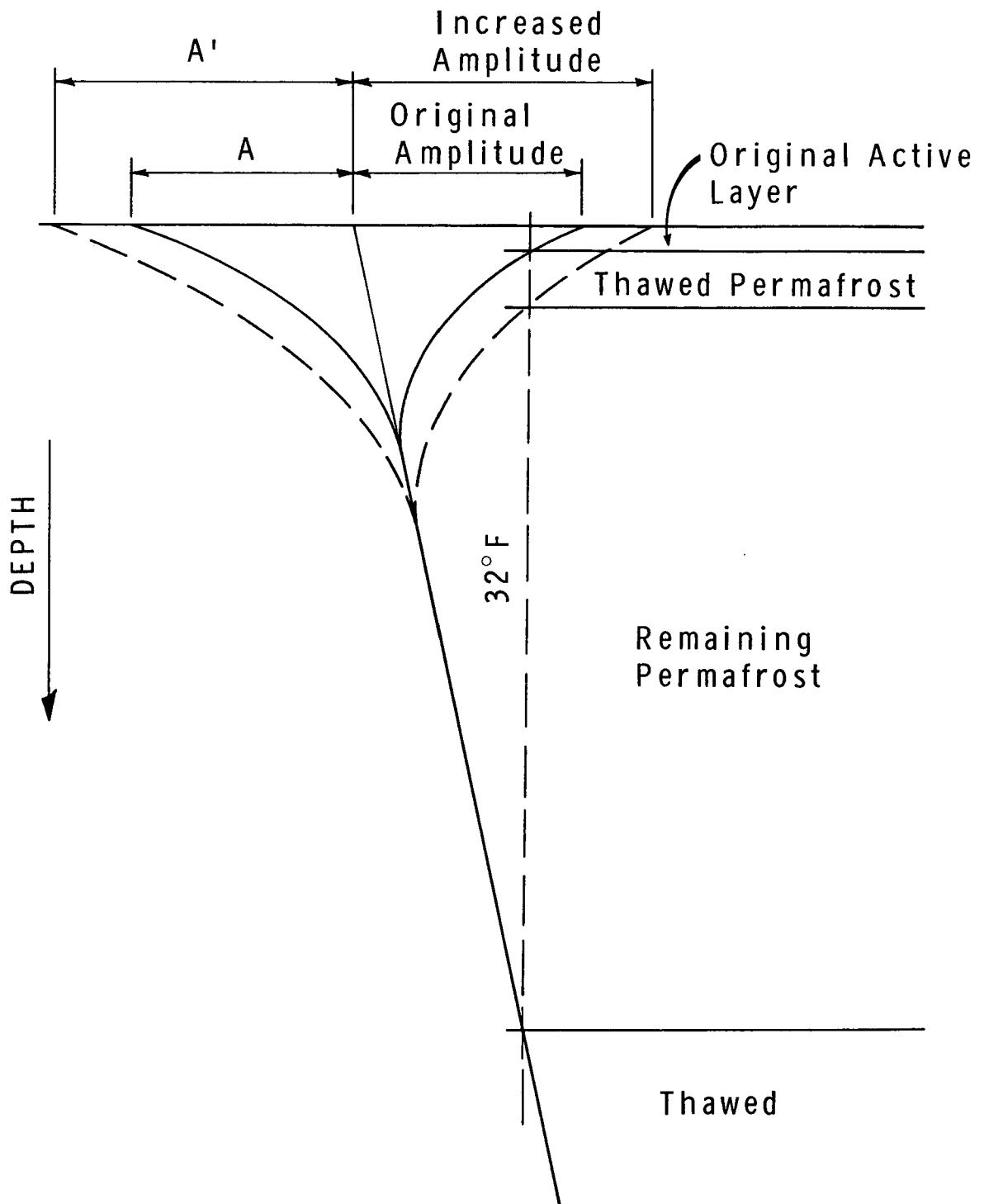


FIGURE 3  
 PERMAFROST THAWING BY INCREASING AMPLITUDE OF  
 SURFACE VARIATION

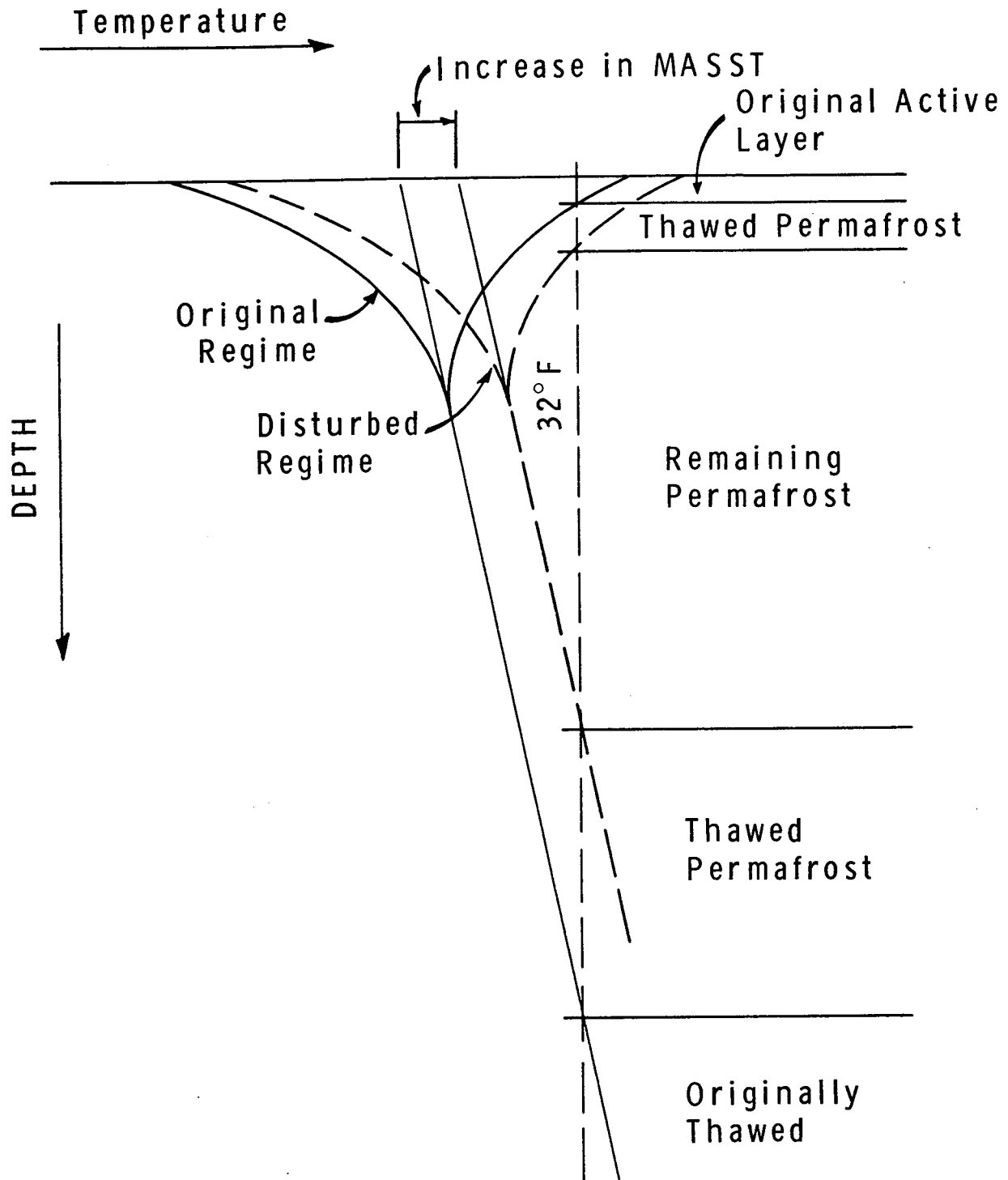


FIGURE 4

PERMAFROST THAWING BY INCREASING THE  
MEAN ANNUAL SOIL SURFACE TEMPERATURE

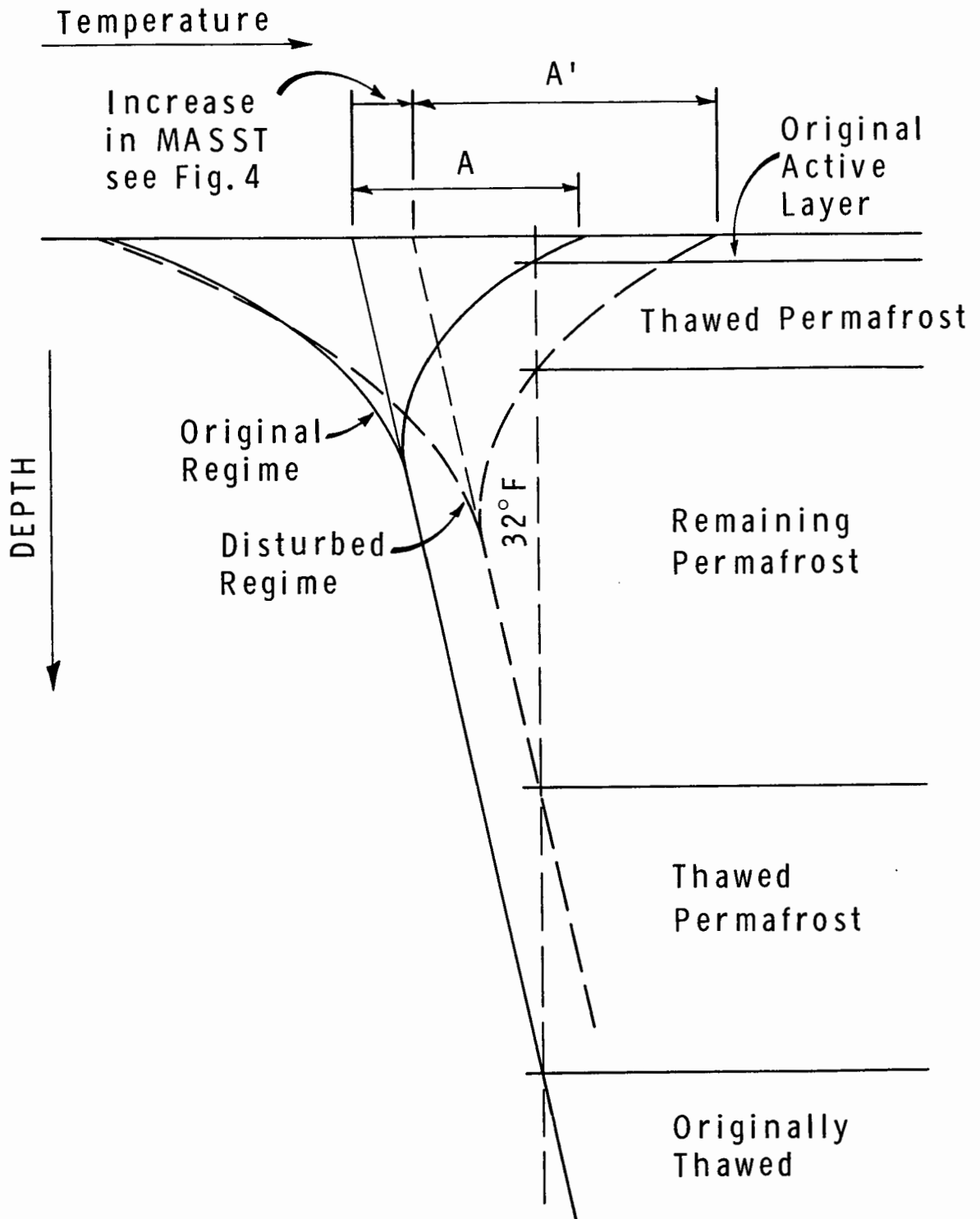
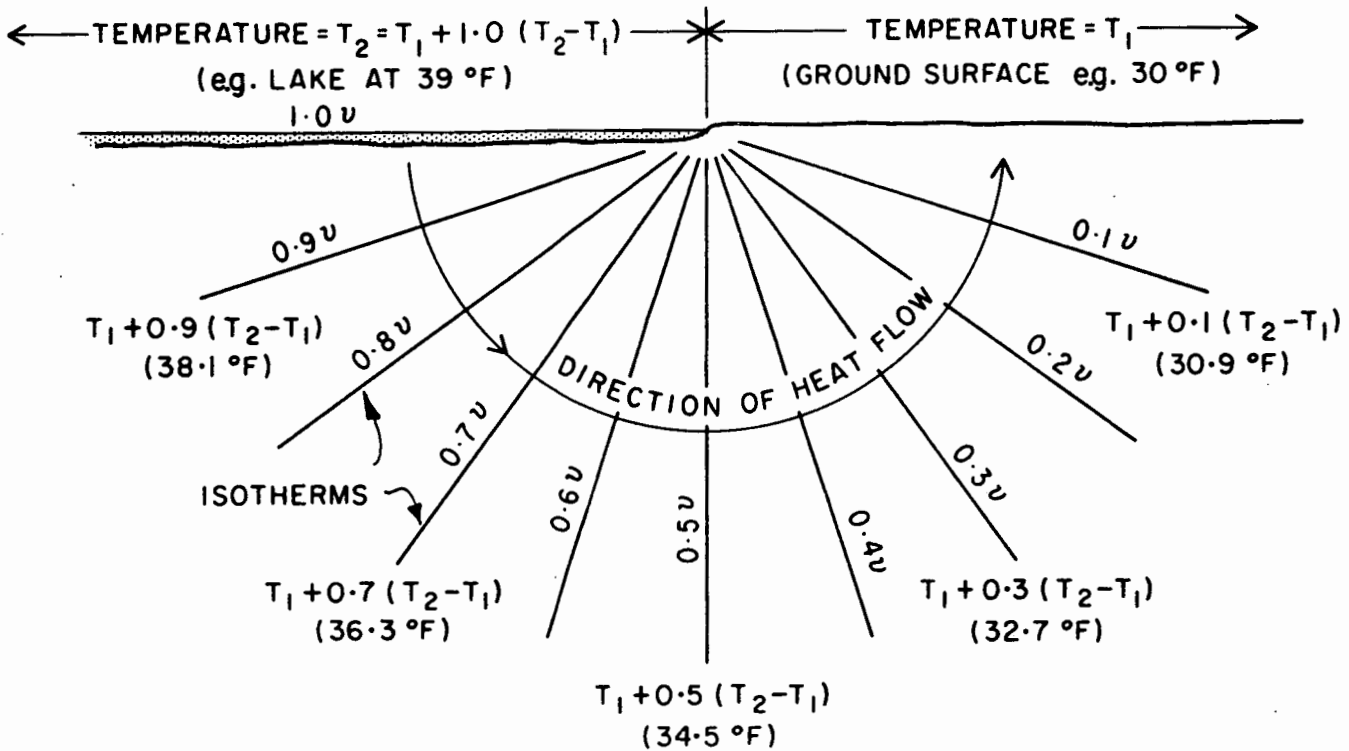


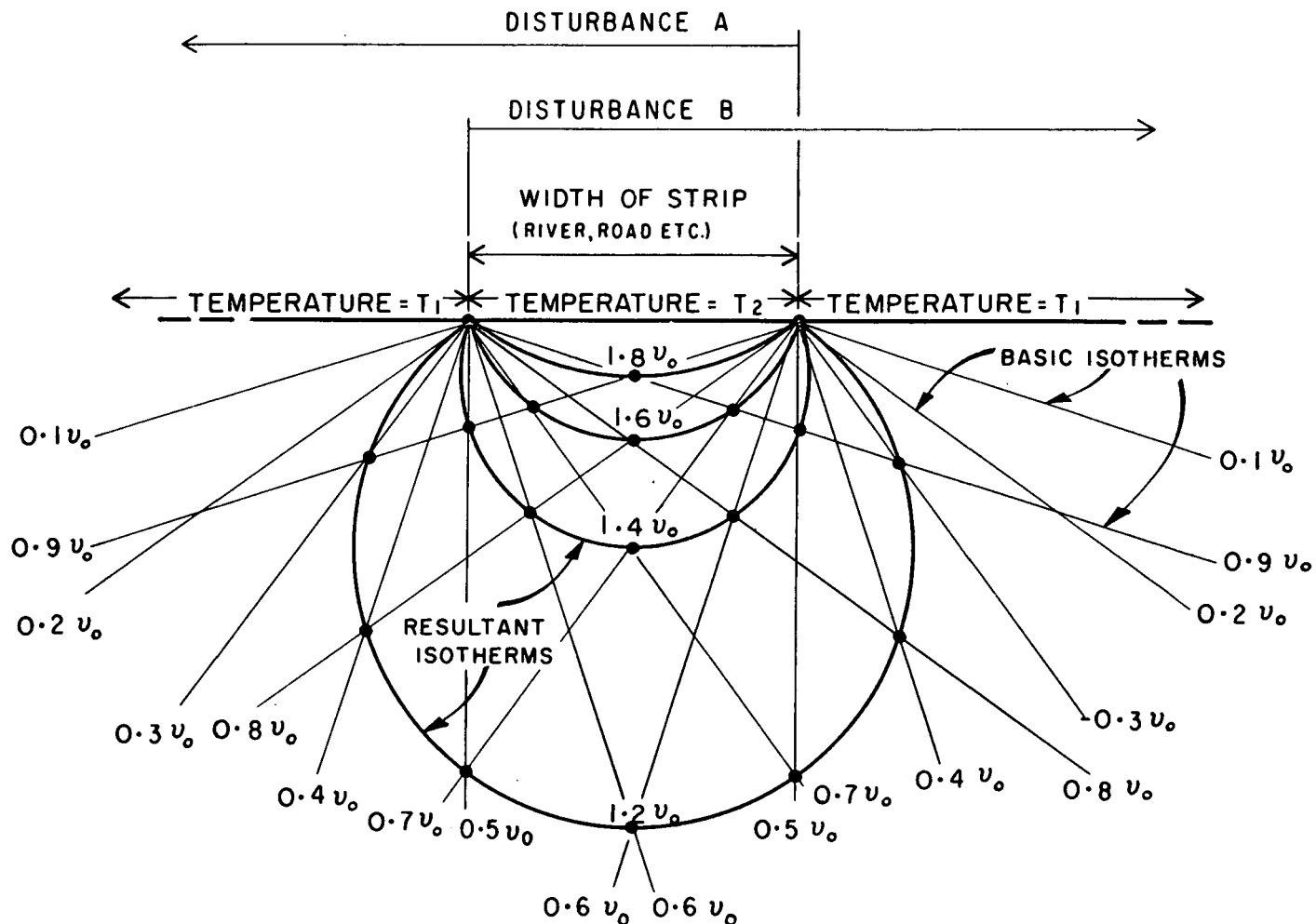
FIGURE 5  
 PERMAFROST THAWING BY INCREASING AMPLITUDE  
 AND AVERAGE SURFACE TEMPERATURES



$$\begin{aligned}
 v &= T_2 - T_1 = \text{The Disturbance} \\
 &= 39 - 30^\circ\text{F} = 9^\circ\text{F}
 \end{aligned}$$

FIGURE 6

THE STEADY TEMPERATURE UNDER THE STRAIGHT SIDE OF  
 A LARGE AREA ON THE GROUND SURFACE. (EXAMPLE: A  
 LAKE AT 39°F IN A REGION WHERE THE MEAN ANNUAL  
 TEMPERATURE AT THE GROUND SURFACE IS 30°F)  
 (After Brown, 1963)



$$2v_0 = \text{Disturbance Caused by Each of the Two Superimposed Problems as in Fig. 6} \\ = T_2 - T_1$$

Note that One "Extra" Disturbance Has Been Added to the System and Must be Subtracted Out, (i. e.  $1.8v_0 - 1.0v_0 = 0.8v_0$ ).

FIGURE 7

THE STEADY TEMPERATURE UNDER A LONG STRIP AREA ON THE GROUND SURFACE: OBTAINED BY DIRECT ADDITION OF THE TEMPERATURES UNDER TWO LARGE AREAS WHOSE EDGES ARE PARALLEL TO ONE ANOTHER. (After Brown, 1963)

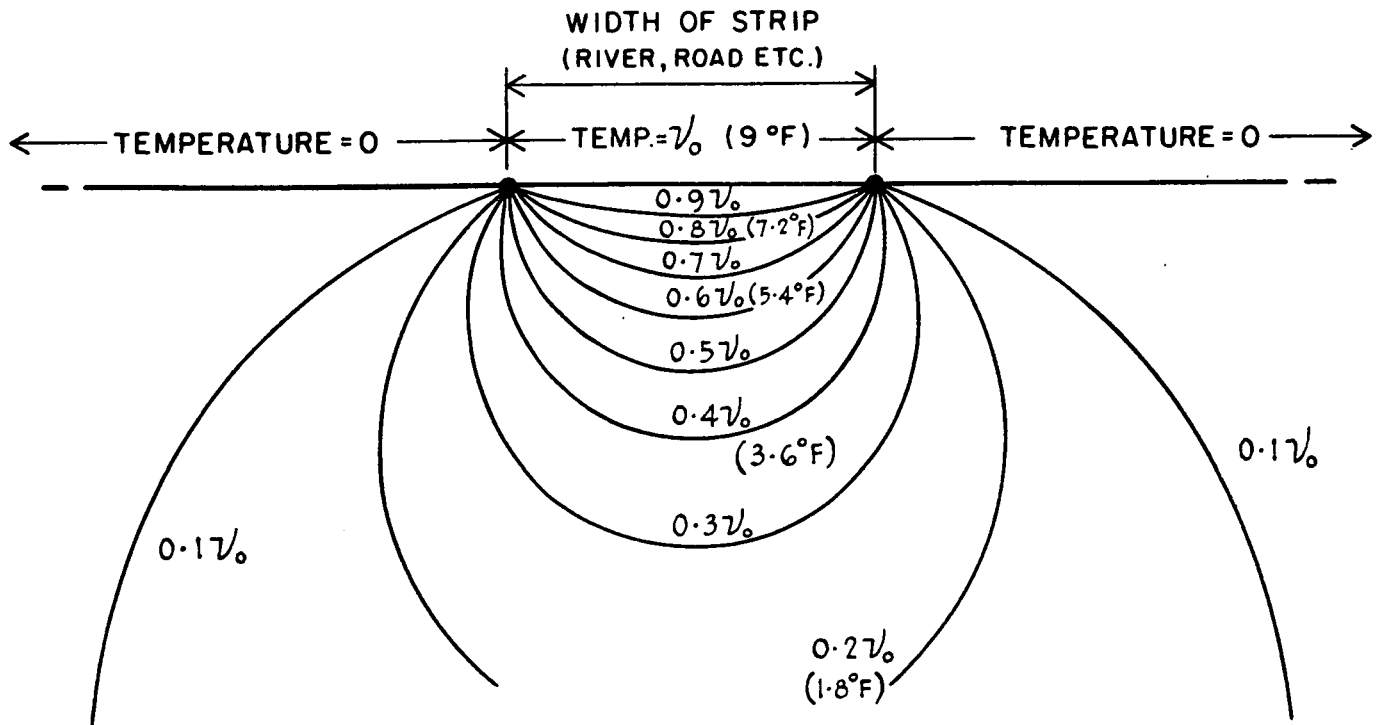


FIGURE 8

THE STEADY TEMPERATURES UNDER A LONG STRIP AREA EXPRESSED AS FRACTIONS OF THE TEMPERATURE DIFFERENCE BETWEEN THE SURFACE OF THE STRIP AND THE REMAINING GROUND SURFACE. (THE SITUATION CORRESPONDS TO A LONG BUILDING, STREET, OR RIVER.) (After Brown, 1963)

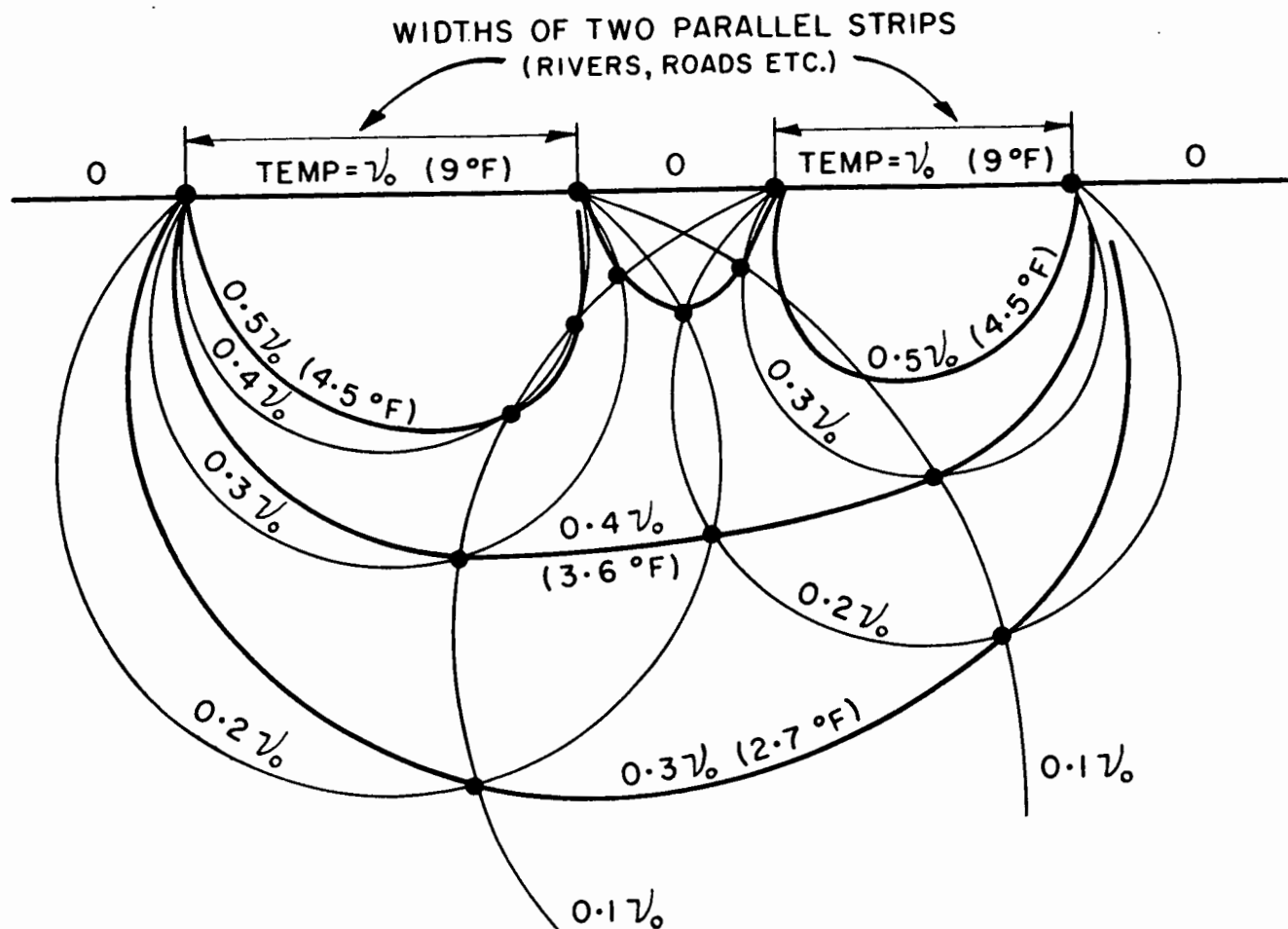


FIGURE 9

THE STEADY TEMPERATURE REGIME IN THE GROUND UNDER TWO PARALLEL STRIPS OF DIFFERENT WIDTHS WHOSE SURFACES ARE MAINTAINED AT A TEMPERATURE OF  $70$  HIGHER THAN THE SURROUNDING GROUND SURFACE. (THE SITUATION CORRESPONDS TO TWO PARALLEL ROADS, RIVERS, OR LONG BUILDINGS.) (After Brown, 1963)

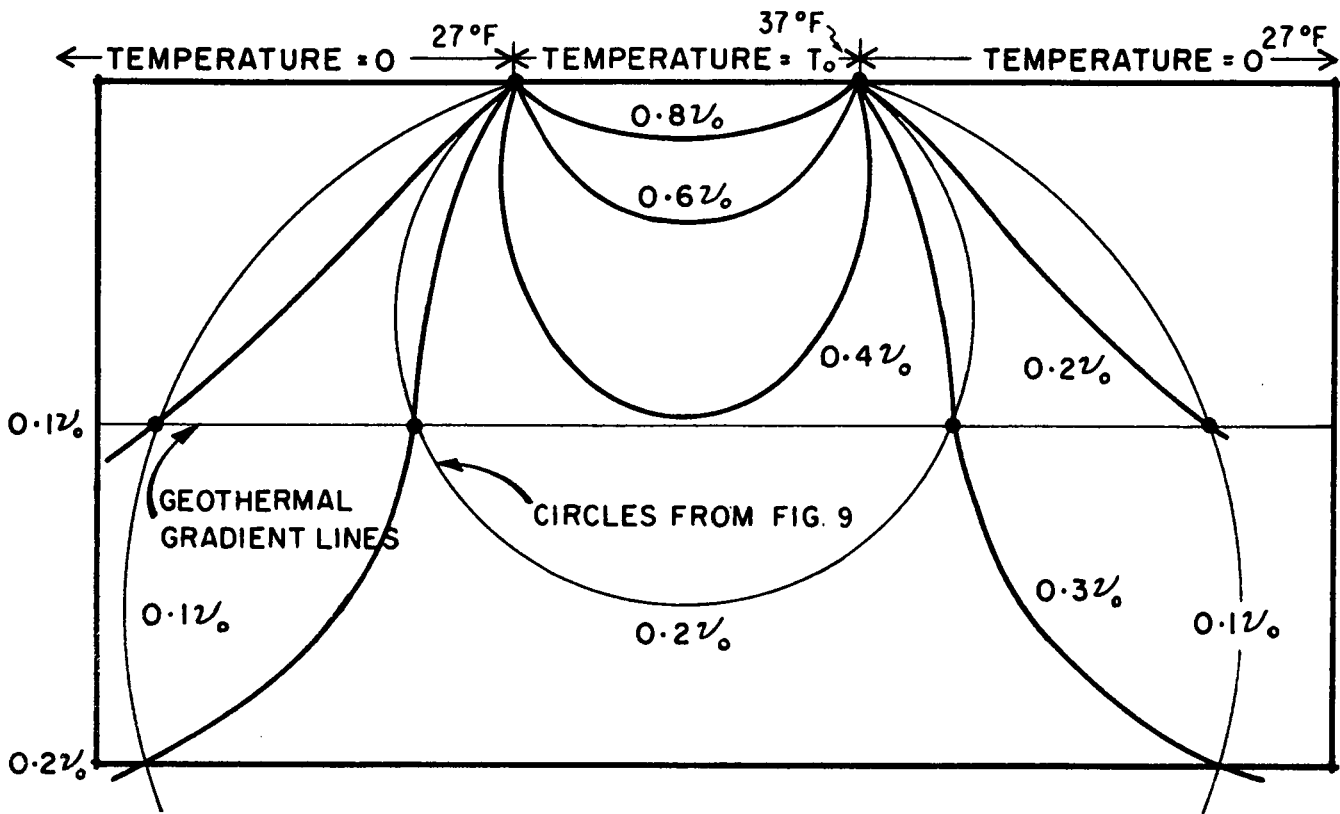


FIGURE 10

STEADY TEMPERATURE UNDER A RIVER, ROAD, OR LONG BUILDING 100 FT WIDE WITH  $\Delta T_0 = 10^{\circ}\text{F}$ , AND A GEOTHERMAL GRADIENT OF  $1^{\circ}\text{F}/100\text{ FT}$ . (After Brown, 1963)

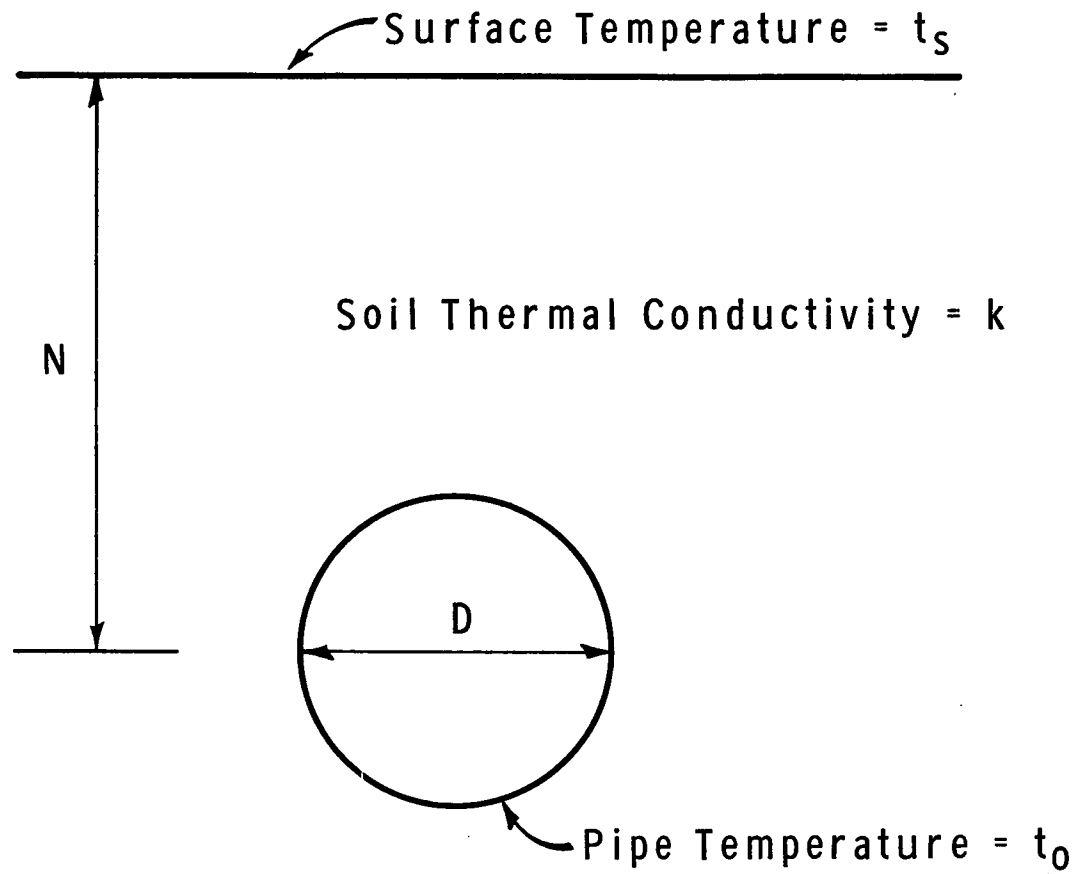
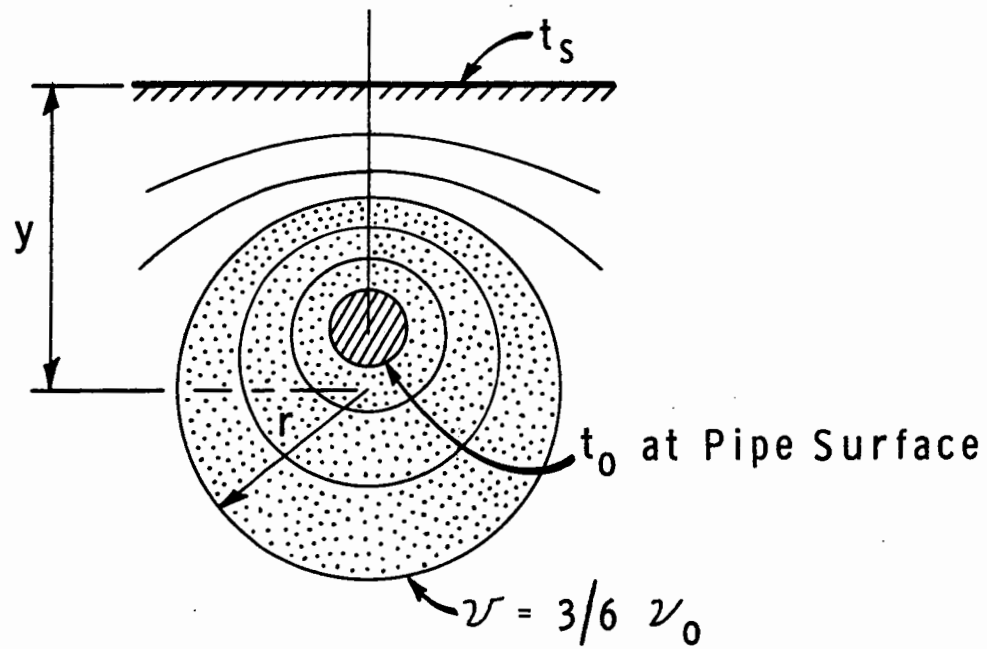


FIGURE 11  
BURIED PIPE NOMENCLATURE

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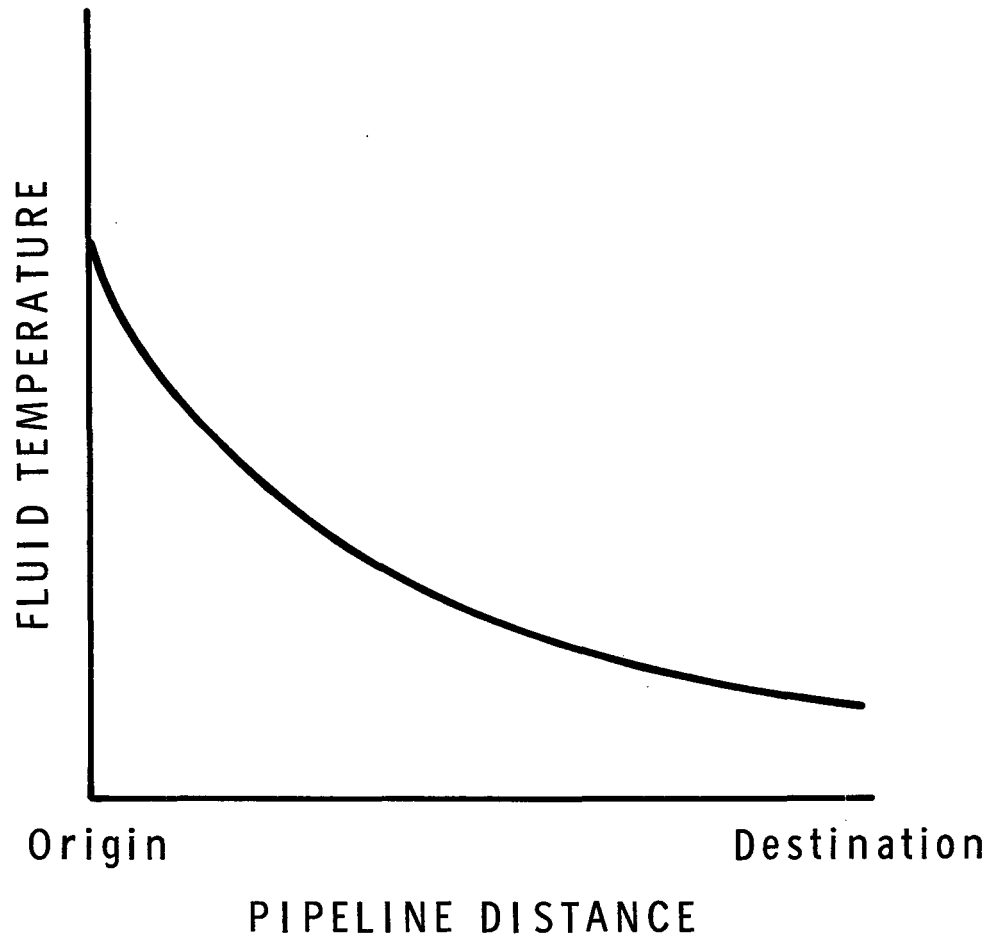
$$\mathcal{V}_0 = t_0 - t_s$$

$$\mathcal{V} = t - t_s$$

For a pipe surface temperature of  $42^\circ\text{F}$  and a frozen soil temperature of  $32^\circ\text{F}$ , the dotted area represents the thaw bulb.

FIGURE 12  
BURIED PIPE ISOTHERMS

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This sketch illustrates the type of results which can be calculated stepwise determining the heat loss through the soil from a buried pipeline. The stepwise increment should be short with large temperature differences, but can be increased toward the end of the pipeline.

FIGURE 13

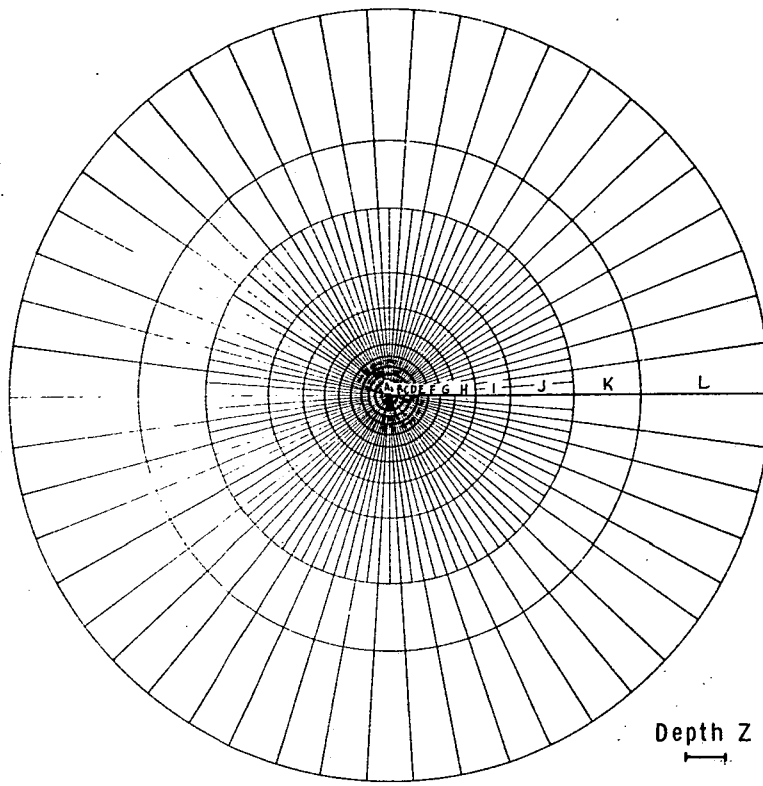


FIGURE 14a  
 FULL CHART BY LACHENBRUCH FOR CALCULATING  
 THERMAL DISTURBANCE.

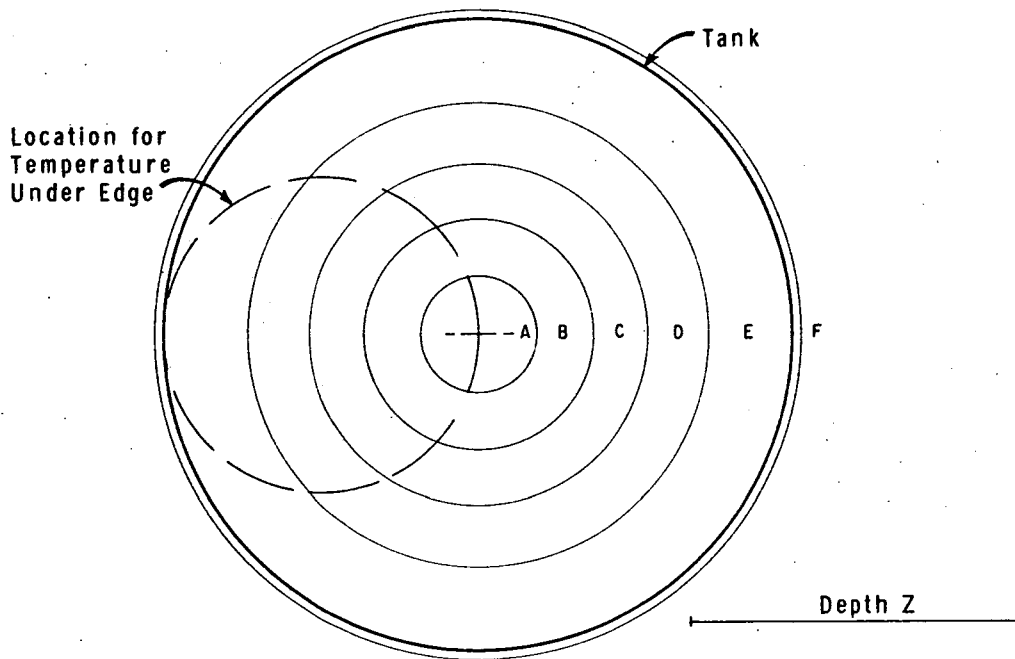


FIGURE 14b  
 EXPANDED SECTION OF FIGURE 14a SHOWING SIZE  
 AND LOCATION OF 200 FOOT DIAMETER TANK.  
 (See Text)

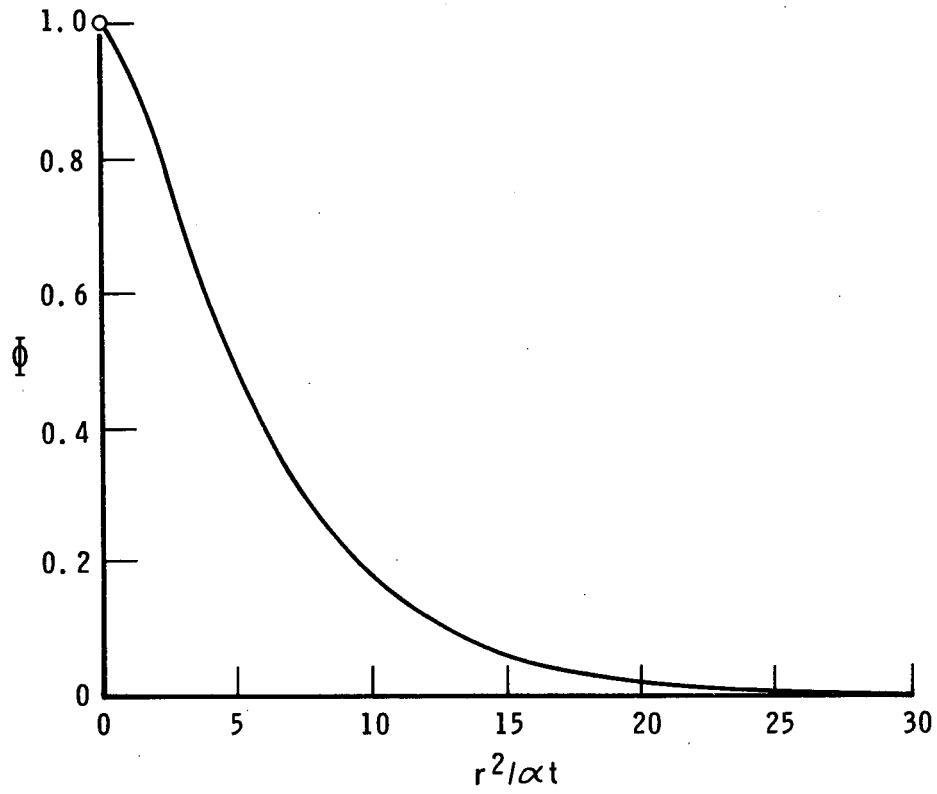


FIGURE 15  
WEIGHTING FACTOR FOR INITIAL DISTURBANCE  
(After Lachenbruch)

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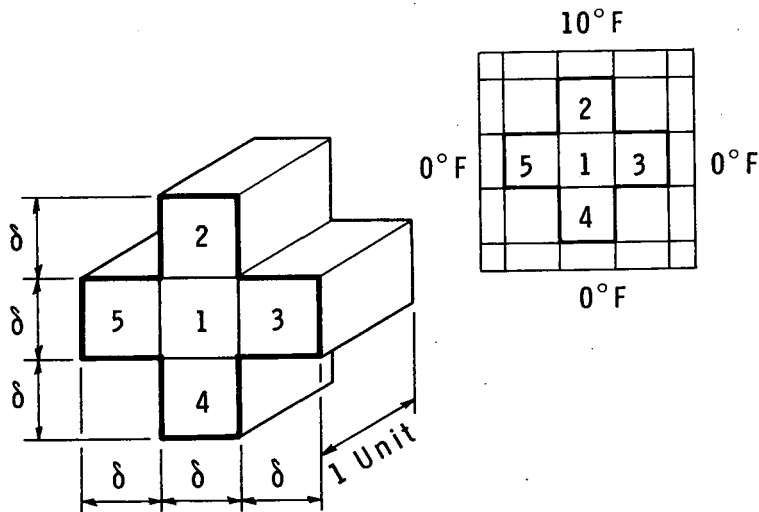


FIGURE 16 RELAXATION GEOMETRY

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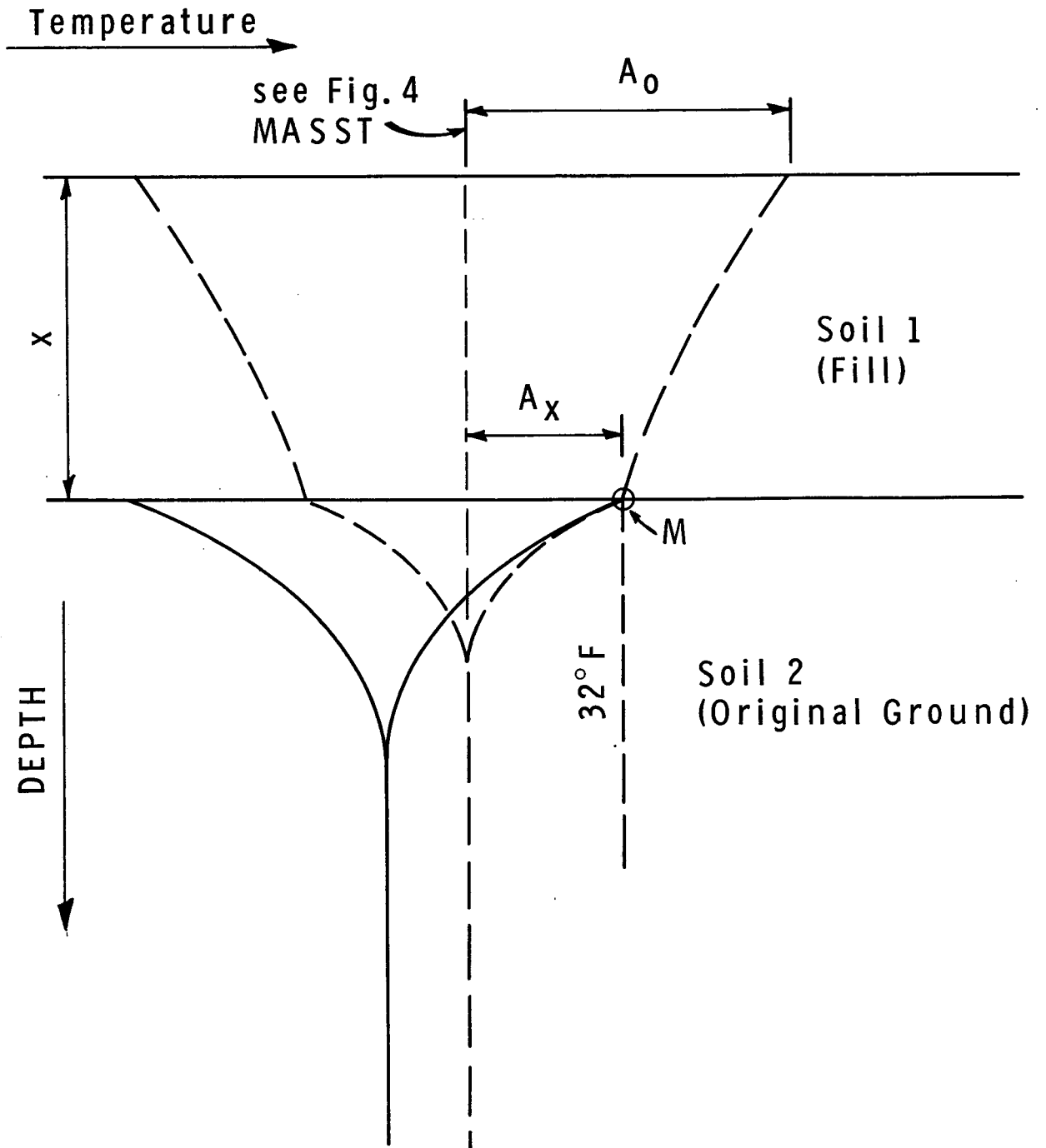


FIGURE 18

FILL THICKNESS REQUIREMENT

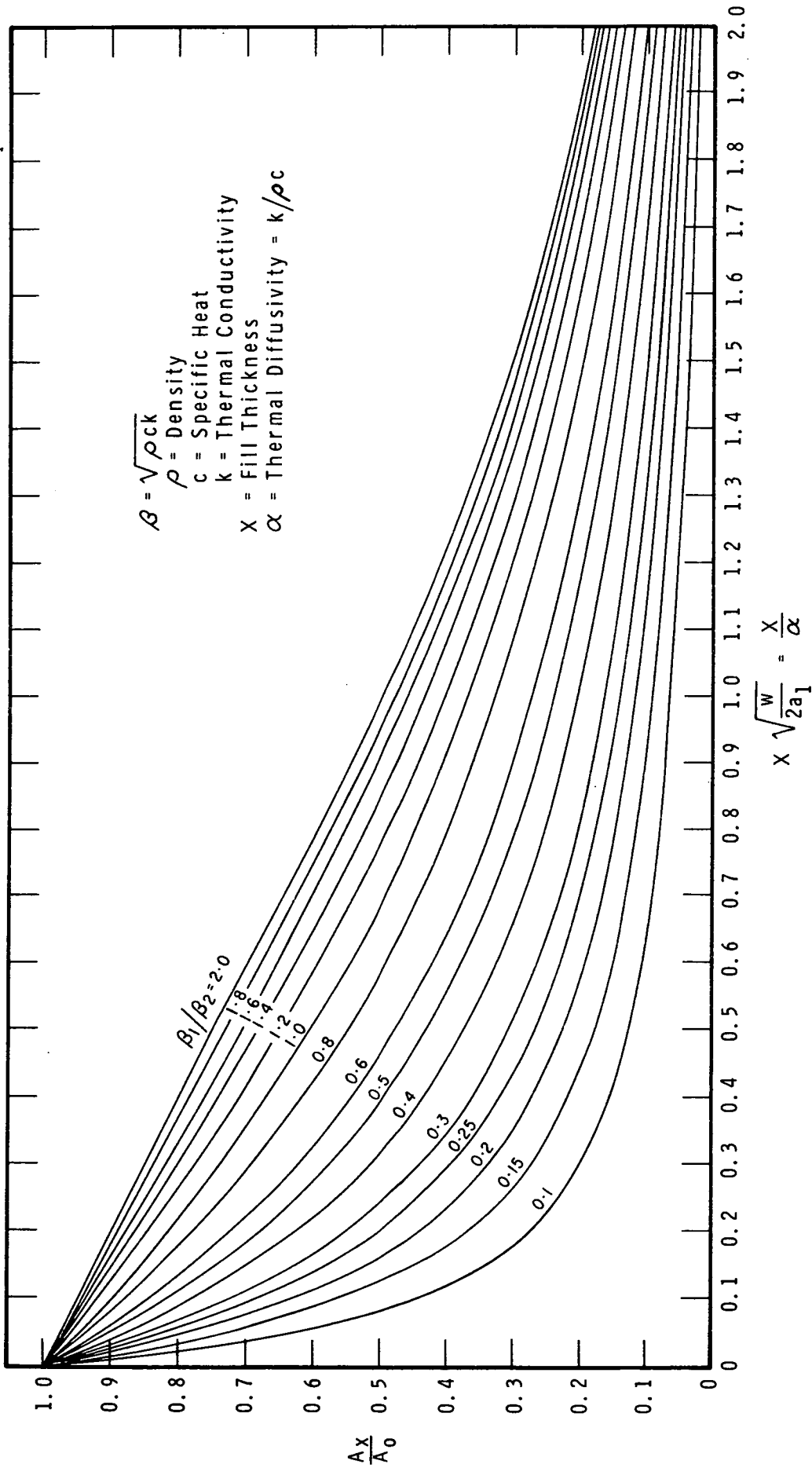


FIGURE 19  
 TWO-LAYER CASE. EFFECT OF THERMAL PROPERTIES AND THICKNESS OF SURFICIAL  
 LAYER ON AMPLITUDE AT THE INTERFACE (AFTER A. H. LACHENBRUCH)

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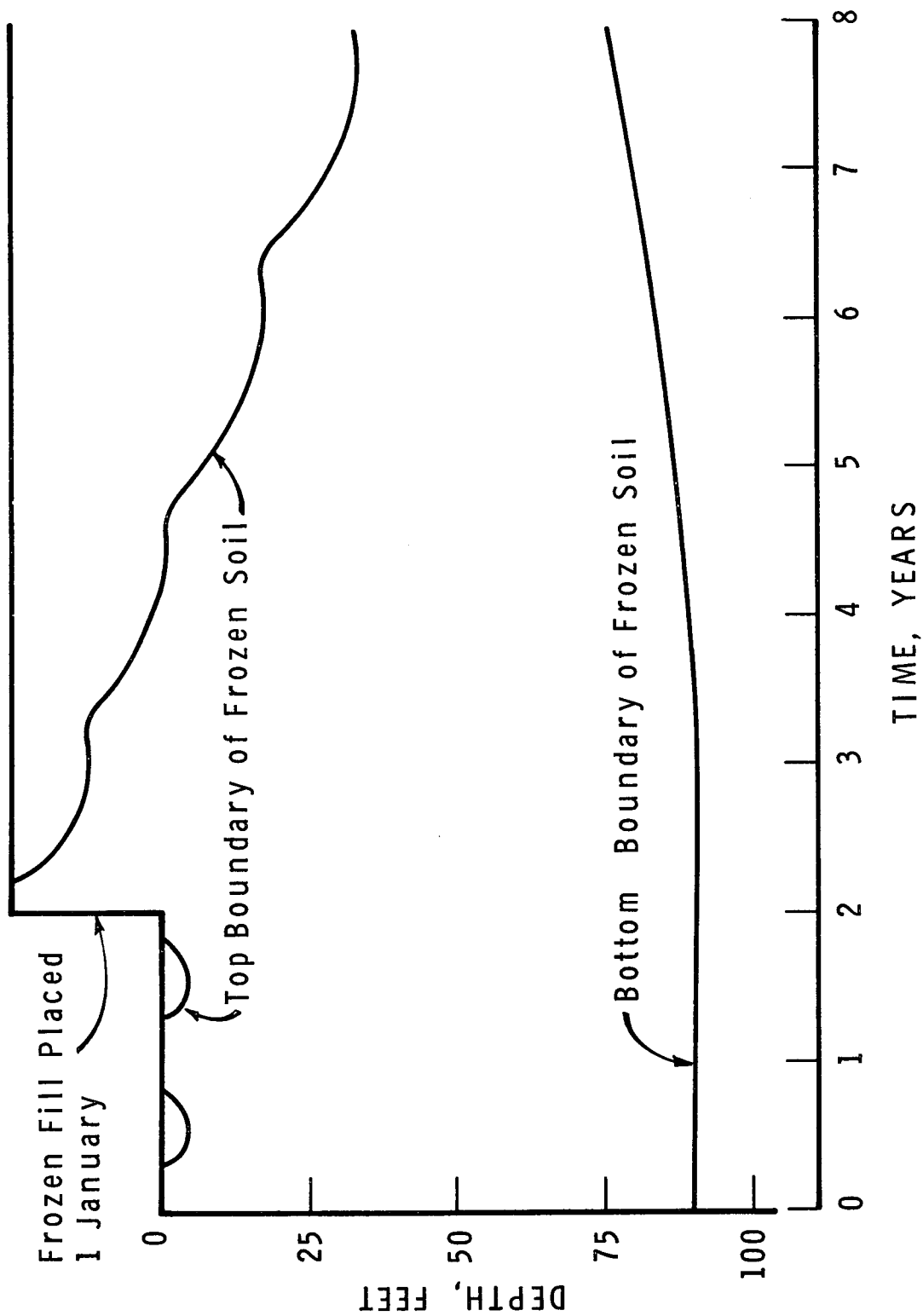


FIGURE 20  
SKETCH OF REPRESENTATIVE ITERATIVE SOLUTION

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TABLE 1

## EQUILIBRIUM TEMPERATURE

1	2	3
Zone	Number of $\Omega$ units per zone	Number of $\Omega$ units subtended by tank
A	20	20
B	40	40
C	60	60
D	80	80
E	100	93
F	100	0
G	100	0
H	100	0
I	100	0
J	100	0
K	50	0
L	50	0
		$\Sigma \Omega = 293$

$$\text{Disturbance} = \left( \frac{293}{1000} \right) (30 \text{ } ^\circ\text{F}) = 8.8 \text{ } ^\circ\text{F}$$

$$\text{Temperature} = 10 \text{ } ^\circ\text{F} + 8.8 \text{ } ^\circ\text{F} = 18.8 \text{ } ^\circ\text{F}$$

Add 2° F for Geothermal Gradient

$$\text{Temperature at Equilibrium} = \underline{\underline{20.8 \text{ } ^\circ\text{F}}}$$

TABLE 2

## TEMPERATURE AT ONE YEAR

Zone	$\Omega$ Units Subtended	$1 + \left(\frac{\bar{\rho}}{z}\right)^2$	$\frac{z^2}{\alpha t} \left[1 + \left(\frac{\bar{\rho}}{z}\right)^2\right]$	$\phi$	$(\phi)(\Omega)$
		(a)	$= r^2/\alpha t$ (b)		
A	20	1.010	24.7	0.01	0.20
B	40	1.080	26.4	0.005	0.20
C	60	1.204	29.6	0.002	0.12
D	80	1.415	34.6	0.001	0.08
E	93	1.782	43.6	0.0	-
F	0	2.383	-		
G	0	3.347	-		
H	0	5.042	-		
I	0	8.477	-		
J	0	17.27	-		
K	0	34.33	-		
L	0	70.26	-		

$$\Sigma = 0.60$$

(a) After Lachenbruch  $\bar{\rho}$  is the average radius of each zone.

(b) "r" is the average distance from the point being studied at depth z to a "rectangular" unit for each zone.

$$z^2/\alpha t = (3048 \text{ cm})^2 / 3.8 \times 10^5 \text{ cm}^2 = 24.4$$

(c) From Figure 15.

Disturbance =  $\left(\frac{0.60}{1000}\right)(30^\circ\text{F}) = 0.018^\circ\text{F}$  at the end of one year at 100 feet. This negligible change demonstrates the long time periods to equilibrium for large structures.

TABLE 3

FILL REQUIRED TO PREVENT PERMAFROST MELT

Case	Subgrade Soil	Fill Soil	$A_o$ °F	$A_x$ °F	$A_x/A_o$	$\beta_1$ cal/cm <sup>2</sup>	$\beta_2$ °C sec <sup>1/2</sup>	$\beta_1/\beta_2$
A	Ice laden Silt	Gravel	30	6	0.20	0.055	0.033	1.7
B	Gravel	Gravel	30	6	0.20	0.033	0.033	1.0

Case	X/γ	γ cm	X cm	X ft
A	1.60	290	465	13.2
B	1.85	290	538	21.2

## Discussion

A question was asked about the relation between mean annual air temperature and ground temperature and the possibility of predicting it for various latitudes and locations. The author directed the question to Mr. G. H. Johnston of the National Research Council who commented that his colleague, Dr. R. J. E. Brown, has made considerable investigations on the relation between air and ground temperatures. The situation at the ground surface is rather complex and there is no simple relation between air temperature and mean annual ground surface temperature mainly because of variations in the types of surface. Further research is required to determine mean annual soil surface temperatures under various conditions. Methods are available to estimate the values of ground surface temperatures but actual field measurements are required. Dr. Brown remarked that the mean annual soil surface temperature is very difficult to measure and very few values of this particular parameter are available. It is necessary to measure the temperature at some depth in the ground and relate it to the air temperature. A convenient depth is the level of zero annual amplitude which is at the bottom of the layer of ground affected by the annual air temperature cycle. This level varies from perhaps 30 to 50 feet below the ground surface. This situation is particularly applicable in the continuous permafrost zone where stable ground temperature conditions prevail through considerable depths. At Resolute, N.W. T. in the Canadian arctic archipelago, for example, the mean annual air temperature is  $3^{\circ}\text{F}$  and the ground temperature at the depth of zero annual amplitude is  $9^{\circ}\text{F}$  giving a difference of  $6^{\circ}\text{F}$ . This value is an average figure which can be used in many cases to relate ground and air temperatures although observations throughout northern Canada show a range of differences of  $2^{\circ}$  to  $10^{\circ}\text{F}$ ; however,  $5^{\circ}$  to  $7^{\circ}\text{F}$  is a reasonable average which appears to prevail at many locations. One of the main reasons for the difference between mean annual air and ground temperatures is the snow cover in the winter portion of the total 12 month period which damps out much of the influence of the cold sector of the annual air temperature cycle. Returning to the soil surface temperature it has been found in surface energy exchange studies that it is somewhat higher than the mean annual air temperature and not quite as high as the mean annual ground temperature. Mr. R. A. Hemstock remarked that the average  $6^{\circ}\text{F}$  difference between mean annual air and ground temperatures could be used in the author's calculations if the mean annual air temperature was known at a site.

Professor G. A. H. Lock asked a question which he subsequently submitted in writing about the technique of superposition referred to in his paper: "This is indeed a powerful yet simple tool, useful in many situations, but it is difficult to see how it may be applied to freezing or melting soil systems. As the author is no doubt aware, the criterion for

the validity of superposition is linearity - both of the governing differential equations and the conditions which they must satisfy. For steady state situations, the governing equation of heat condition, the diffusion equation, will be rendered non-linear if the medium (soil) properties are temperature-dependent. Even if the properties are constant, radiation (according to the Stefan-Boltzmann Law) at a boundary and differences between the properties of frozen and unfrozen material still invalidate superposition, at least in principle. In addition to the above-mentioned difficulties for steady systems, the corresponding unsteady systems have yet a further non-linearity associated with them. This is due to the movement of the interface between frozen and unfrozen material and is therefore an inherent feature. The author is correct in stating that heat transfer problems in moist soil are soluble but it is not at all certain that they are soluble by such elementary means. On occasion, departures from linearity may perhaps be small but the circumstances under which this is true are complex and have yet to be delineated. I am sure the author will agree that approximate solutions are only valuable to the extent that the degree of approximation is known."

Dr. Peyton replied that a high degree of precision is not necessary and it is sufficient to calculate the depth of thaw within one or two feet. Pile lengths and building settlements are not going to be ascertained precisely. He agreed with many of the points raised by Professor Lock but he did not feel it was appropriate to present a lengthy discussion of the complexities of the calculations. Superposition is possible if isotropic homogeneous soil conditions are assumed and only the steady state case is considered. This permits an examination of rather complicated problems with fairly simple calculating techniques. Of course, many difficulties arise if the soil is not isotropic and homogeneous. All the steady state problems were based on the mean annual soil surface temperature and all surface heat losses, considered in the Stefan-Boltzmann Law including the fourth power law for radiation, were ignored. Dr. Peyton added that the reasons for the long involved numerical calculations in his last example were related to the problems of the non-linear heat diffusion problems, changes of soil properties with temperature and between the frozen and unfrozen state. Second, third and fourth power non-linear partial differential equations are introduced, none of which has yet been solved. The depth of thaw for one, five, ten, infinity years, was not solved by superposition but with another technique from one of Lachenbruch's papers which is a dynamic rather than a static solution. This was used to demonstrate that these problems are solvable.

Mr. W. Quinn added a comment to the discussion on the differences between mean annual air and ground temperatures. The determination of the surface boundary condition is felt by the U. S. Army Terrestrial Sciences Center to be very important. Dr. R. Scott of the

California Institute of Technology has been carrying out some work for the Center to establish a correlation by considering such factors as insolation and convective effects. He agreed with Mr. G.H. Johnston that the time has arrived to attempt some sort of field correlation. He cited the problem at Thule, Greenland where the runway, which had a black asphalt surface, was experiencing unexpected subsidence. A decision was made to paint it white which changed the surface boundary condition thus reducing the depth of thaw by 2 feet. Dr. Peyton remarked that other problems may arise in this sort of situation. For example, the white runway surface at Thule produced severe ice conditions making aircraft landings very hazardous. Thus the solution to a problem is rarely straightforward. Mr. Quinn replied that the icing problem appeared to be a matter of opinion because reports he had received indicated that it was not significant. Dr. Peyton commented on Dr. Scott's work. The U. S. Army Terrestrial Sciences Center (formerly the Cold Regions Research and Engineering Laboratory) at Hanover, New Hampshire, published an engineering science bulletin which encompasses much of Scott's early work correlating atmospheric conditions, snow cover, vegetation and other factors to soil surface temperature. His work is excellent but Dr. Peyton did not mention it because it was not applicable to his presentation. There are still gaps in Scott's work in terms of determining the proper thickness of a fill for example. Dr. Peyton wished to reinforce the statement by Dr. Brown regarding the mean annual soil surface temperature, that it is possible to obtain a useful value simply by drilling a hole and taking the temperature of the ground below the level of seasonal fluctuations. If the type of soil is known, it is possible to calculate the depth of zero annual amplitude fairly accurately, drill to this depth, and measure the temperature without large error.