A comparison of triaxial and plane strain tests on frozen silt

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The results of triaxial and plane strain tests on a dense, saturated frozen silt at − 10°C are reported. Both deformation rate-controlled tests and constant stress creep tests under zero minor principal stress were carried out. Stress conditions at failure in rate-controlled tests and critical stress conditions in creep tests indicate that a von Mises failure criterion can be used to relate unconfined triaxial and plane strain failure conditions.

The application of elastic equations to stress analysis is discussed and it is noted that the intermediate principal stress does not attain a value corresponding to elastic predictions. It is also noted that the deformation modulus calculated from elastic equations is considerably higher in plane strain tests and that the strain rates are strain dependent, attaining the ideal no-volume change value (Poisson's ratio = 0.5) for a saturated media only at or near critical stress levels.

Dans le présent article, sont présentés les résultats d'essais de déformation biaxiale et triaxiale, menés sur un silt dense gelé, à une température de − 10°C. On a effectué à la fois des essais de déformation à vitesse contrôlée et des essais de fluage sous contrainte constante, en présence d'une contrainte principale mineure égale à zéro. Les conditions de contrainte au niveau de rupture lors des essais à vitesse contrôlée, et les conditions de contrainte critiques lors des essais de fluage, indiquent que l'on peut employer le critère de rupture de von Mises pour corréler les conditions de rupture par déformation biaxiale et déformation triaxiale sans confinement.

On commente l'application des équations d'élasticité à l'analyse des contraintes, et l'on note que la contrainte principale intermédiaire n'atteint pas une valeur correspondant aux valeurs d'élasticité prédites. On note aussi que le module de déformation calculé au moyen des équations d'élasticité est beaucoup plus élevé dans les essais de déformation biaxiale, et que les vitesses de déformation sont fonction de la déformation et atteignent la valeur idéale, avec une augmentation nulle de volume (coefficient de Poisson = 0,5), dans le cas d'un milieu saturé soumis uniquement à une contrainte critique ou presque critique.

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Introduction

Most laboratory studies on frozen soils have been conducted under radially symmetric conditions and the common stress condition is triaxial compression where $\sigma_1 > \sigma_2 = \sigma_3$. Many field problems involve boundary conditions approaching plane strain and it is usual that $\sigma_1 > \sigma_2 > \sigma_3$. There is a need to examine whether classical stress-analysis techniques can be applied to frozen soils in bridging differences between laboratory and field conditions.

Material and Equipment

Triaxial and plane strain test samples of frozen South Nation River silt (effective grain-size $D_{10} = 0.02$ and uniformity coefficient $D_{60}/D_{10} = 2.3$) were prepared by quick freezing in a consolidated (porosity = 0.37) saturated state. Both constant deformation rate tests and constant stress creep tests were carried out under triaxial and plane strain conditions at − 10°C (±0.5°C).

The triaxial equipment was conventional except for the volume-change measuring equipment which has been described by O'Connor and Mitchell (1978*a*). The plane strain apparatus (Figure 1), is essentially a steel vise equipped with a load cell to monitor the intermediate principal stress. Friction was reduced by using teflon sheets on the sides of the samples and a thin layer of grease on the ends. Two dial gauges mounted on the base of the apparatus monitored the

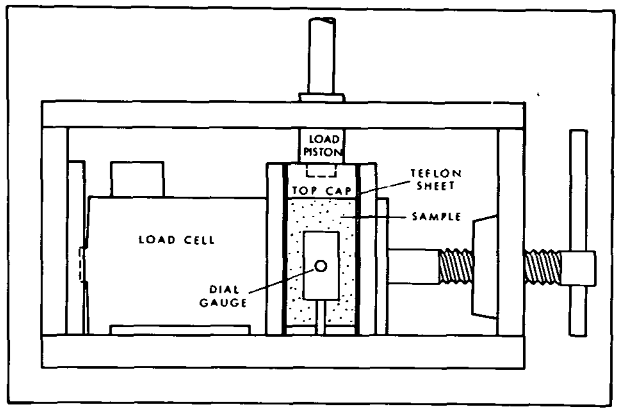

FIGURE 1. Plane strain test equipment for frozen soil.

lateral displacements of small steel pads attached to the minor principal plane boundaries of the sample. These boundaries appeared to remain planar and parallel throughout the pre-failure deformation of samples and the measured displacements were used to calculate lateral strains. Load adjustments were made, in all creep tests, to maintain a constant axial stress in the test samples.

Test Results

Displacement rates of 0.01 to 1 mm/min were used in comparative unconfined triaxial and plane strain tests (equivalent to average strain rates of 10^{-2} to 10^{-4}/min). Figures 2 and 3 show typical data from these tests and test samples exhibited shear failures at approximately 60° to the major principal plane. A tangent modulus, $T_1 = \delta\sigma_1/\delta\varepsilon_1$, constant at about 4.2 $\times$ 10^5 kPa for triaxial and 7 $\times$ 10^5 kPa for plane strain was defined. Further loading produced a secondary modulus, $Ts = \delta\sigma_1/\delta\varepsilon_1$, which decreased slightly as the strain rate was decreased, but averaged 6 $\times$ 10^4 kPa for both triaxial and plane strain tests.

Volume changes, calculated from boundary displacement measurements, were dilative (volume increases) as the maximum shearing resistance was

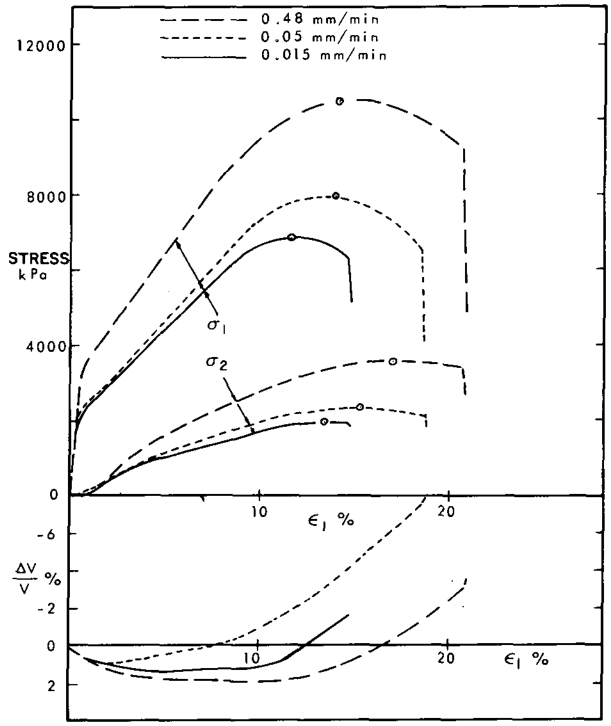

FIGURE 3. Typical stress–strain curves from rate-controlled plane strain tests.

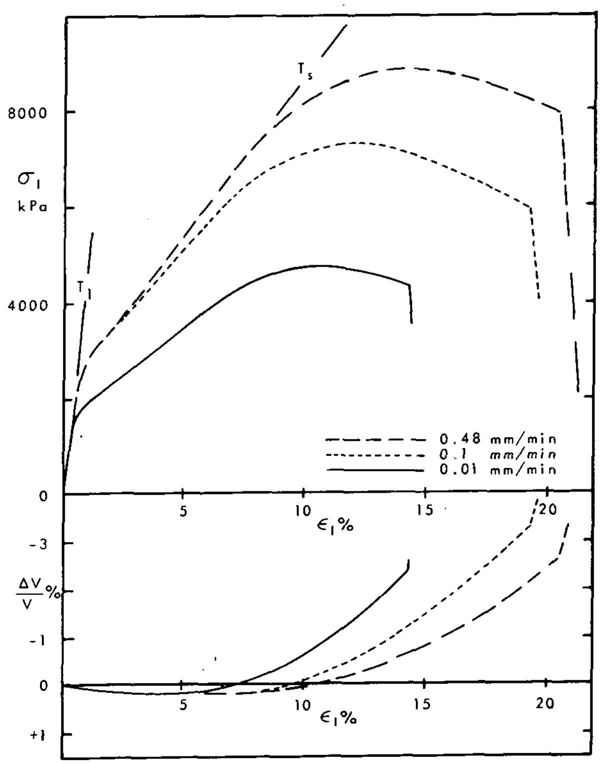

FIGURE 2. Typical stress–strain curves from rate-controlled unconfined triaxial tests.

attained. A strain ratio $\nu = -\delta\varepsilon_3/\delta\varepsilon_1$ was calculated from the triaxial results and was found to vary from between 0.3 to 0.4 during initial tangent loading to close to 0.5 as failure was approached. At failure, rapid and erratic dilation ($\nu > 0.5$) was observed and is considered to be associated with internal cracking of the frozen material. The strain ratios from typical plane strain tests are plotted on Figure 4. Although the samples were placed under a small, lateral seating load prior to testing, the very small initial values of strain ratio might be due to seating in the confined direction. It is significant, however, that axial strains in the order of 3 to 4 per cent were attained before this strain ratio reached close to the value of 1.0 associated with no volume change in plane strain. Again, the erratic dilative behaviour at failure can be noted from Figure 4a. The stress ratio, σ_2/σ_1, at any time during a test can be obtained from Figure 3. This ratio increases linearly from a zero value to a value of about 0.25 at about 4 to 5 per cent axial strain and then increases more slowly, attaining a value of about 0.3 at failure. These values are substantially less than a theoretical elastic value of $\sigma_2 = \nu\sigma_1$ for the $\sigma_3 = 0$ test condition used.

Figure 5 shows the failure values of σ_1, $q = (3/\sqrt{2})\,\tau_{oct}$ and $(q/_p) = (3/\sqrt{2})\tau_{oct}/\sigma_{oct}$ for all rate-controlled tests. Better agreement, between the triaxial and plane

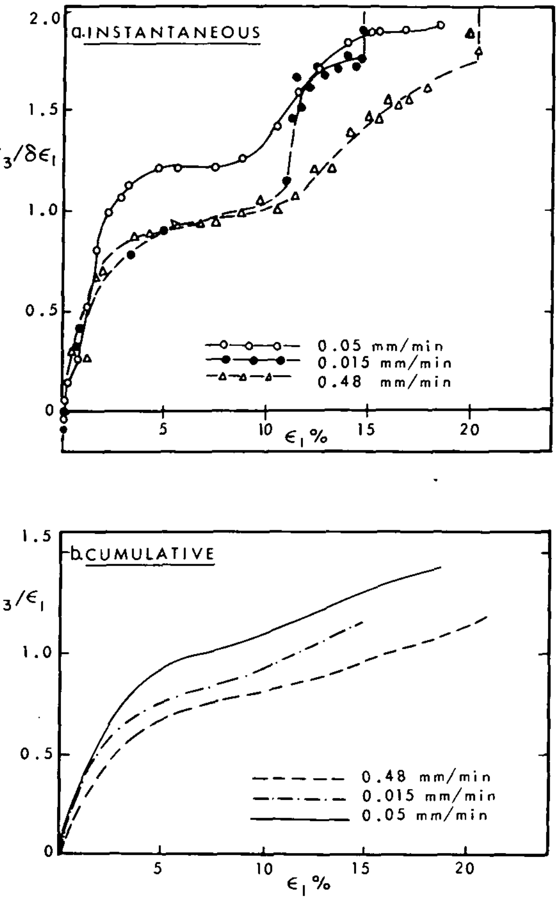

FIGURE 4. Strain ratios during plane strain tests.

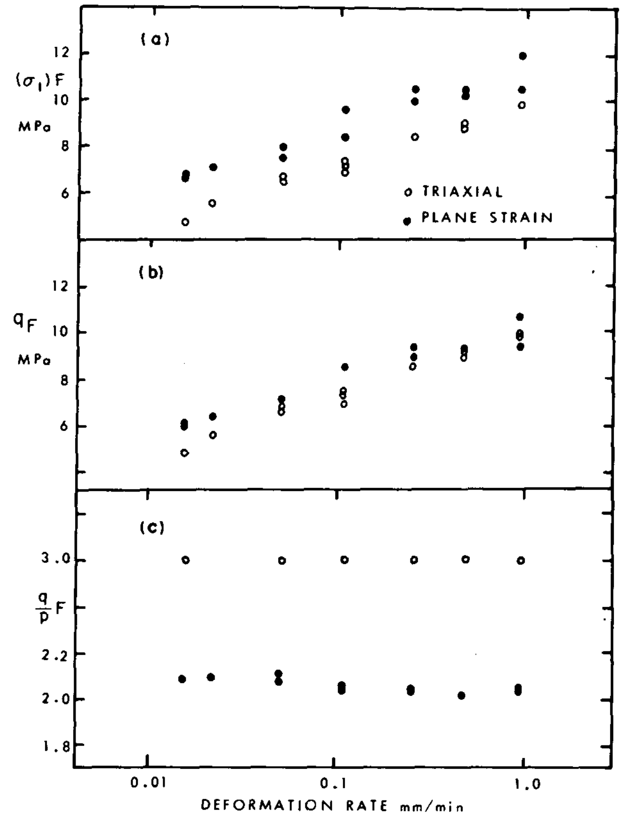

FIGURE 5. Stresses at failure.

strain data, in Figure 5**b** than in Figure 5**a** indicates that a von Mises failure criterion can describe the failure states of frozen soils under low confining stresses. The shear stresses at failure are noted, however, to increase moderately with increased strain rate (in this case, a 30-per cent increase per log cycle of deformation rate increases is apparent). When the shear stresses at failure are normalized by dividing by the mean normal stress $p=\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \sigma_{oct}$, as in Figure 5**c**, the rate dependency is negligable but it appears that the material does not follow an extended von Mises nor an extended Tresca failure criterion (*i.e.* the contribution of the intermediate principal stress to shearing resistance is less than would be anticipated from either of these criterion). Since confined compression tests were not carried out, the applicability of frictional strength criterion cannot be evaluated.

Results from constant axial stress creep tests are plotted on Figures 6 and 7. According to classic creep theory, the secondary phase of creep is characterized by a constant creep rate. These results show that creep rates either continued to decrease with time (low stress levels) or attained a minimum value and thereafter increased until rupture was imminent. Similar behaviour has been reported by Akili (1970) from tests on frozen clay and by Campenella and Vaid (1974) from tests on unfrozen Haney clay. Comparing Figures 6 and 7, it appears that the axial stress required to cause accelerating creep rates for any given critical time is about 25 per cent higher under plane strain than under triaxial stress conditions. Approximately the same values of q, however, exist under both stress conditions for any critical time of accelerating creep rates. Strain ratios in constant stress creep tests were found to follow a similar pattern to those from the deformation rate-controlled tests. The stress ratio σ_2/σ_1 increased rapidly to about 0.25 and then increased more slowly to values of about 0.3 as the creep strain progressed, giving a similar pattern to that found from rate-controlled tests.

Further relationships between the stress-controlled and deformation rate-controlled tests have been discussed by O'Connor and Mitchell (1978*b*) in terms of an energy surface concept. The discussion and con-

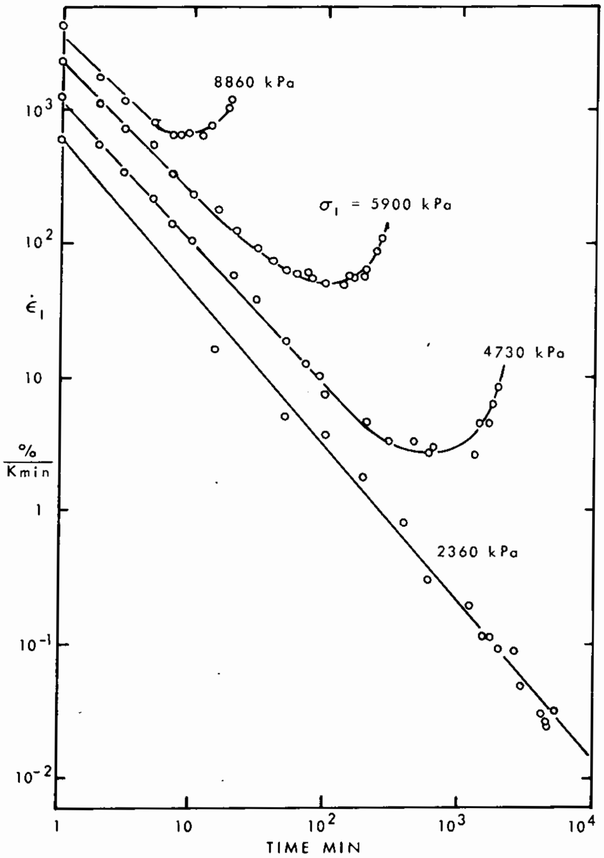

FIGURE 6. Strain rate curves for triaxial creep tests.

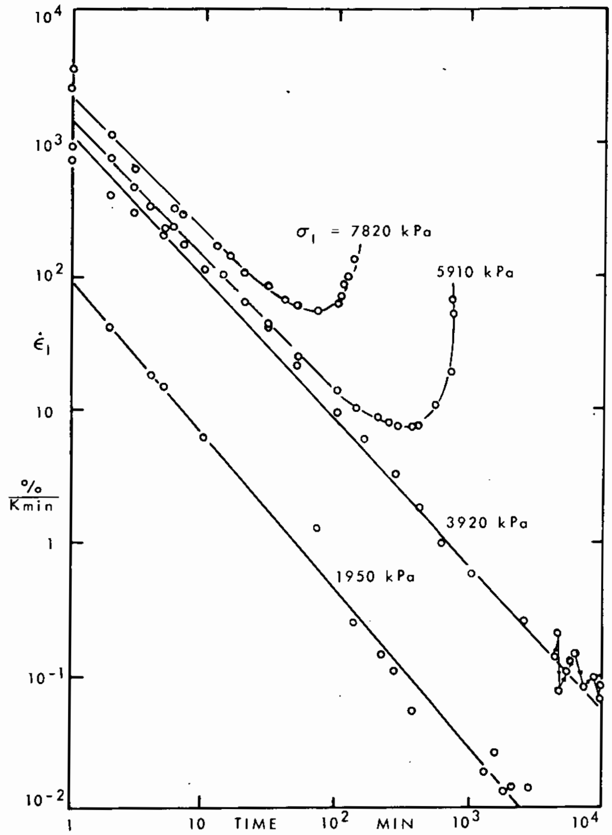

FIGURE 7. Strain rate curves for plane strain creep tests.

clusions which follow are concerned only with the use of pseudo-elastic parameters to describe the stress–strain behaviour of frozen soil.

Discussions and Conclusions

While it is well known that frozen soil does not behave as an elastic media, it is often assumed that pseudo-elastic parameters (in an incremental form) can be applied in approximate analysis of complex behaviour. For plane strain conditions the common incremental elastic constants can be written, for $\delta\sigma_3 = 0$, as

$$[1] \qquad E_i = \frac{\delta\sigma_1}{\delta\varepsilon_1}(1 - \nu^2) = \frac{\delta\sigma_1 + \delta\sigma_2}{\delta\varepsilon_1 - \delta\varepsilon_3}$$

$$[2] \qquad \nu_i = \frac{\delta\sigma_2}{\delta\sigma_1} = \left(1 - \frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right)^{-1}$$

From equation 2 the condition of no volume change requires that $\delta\varepsilon_1 + \delta\varepsilon_3 = 0$ and $\nu_i = 0.5$. For triaxial conditions $E_i = \delta\sigma_1/\delta\varepsilon_1$ and $\nu_i = -\delta\varepsilon_3/\delta\varepsilon_1 = 0.5$ for no volume change.

Figure 8 presents E_i for all deformation rate-controlled data where $\varepsilon_1 < 0.5$ per cent (initial tangent moduli). There appears to be no systematic variation of E_i with either strain or strain rate but the average value for plane strain is 1.67 times the average value obtained from triaxial tests. Yielding of the soil was found to be dependent on deformation rate but occurred at about 1 per cent axial strain with the sub-

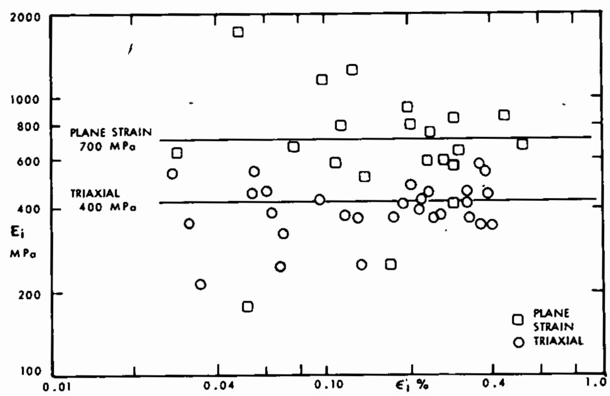

FIGURE 8. Tangent moduli from displacement-controlled tests.

sequent stress–strain relation being rate dependent.

Figure 9 presents values of Poisson's ratio calculated using measured stresses and measured deformations. While there appears to be little variation between stress and deformation rate-controlled tests, it is apparent that the theoretical equality in equation 2 does not hold true for this frozen soil. During all rate-controlled tests the no volume change value of $\nu = 0.5$ coincided approximately with the maximum shearing resistance. Similar values of Poisson's ratio were measured at the critical point (when the strain rate was a minimum) in stress-controlled creep tests. Warder and Andersland (1971) noted a strain-dependent Poisson's ratio in radially symmetric, plane strain (hollow cylinder) tests. They also noted the time lag in build-up of the intermediate principle stress.

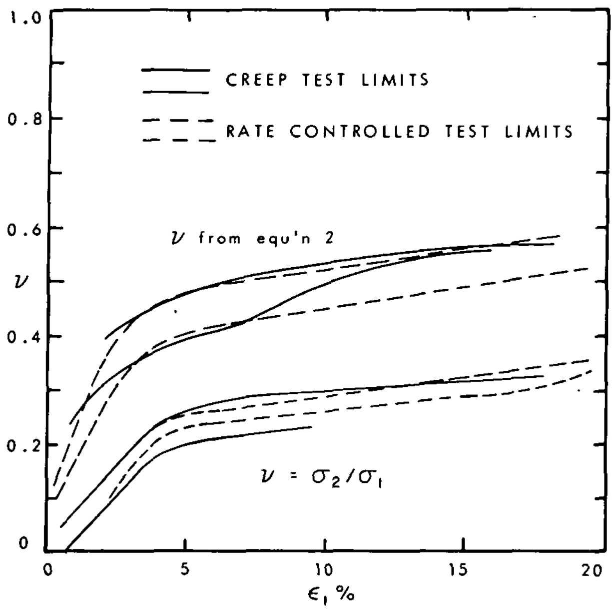

FIGURE 9. Range of Poisson's ratio, ν, from plain strain tests.

The main conclusion drawn from these results is that the application of elastic analysis to estimating stresses and deformations in frozen soil must be treated with caution. Not only are the deformations time dependent but the moduli and strain rates (Poisson's ratio) are dependent on strain and on loading conditions. Variable strain rates and a lag in build-up of the intermediate principle stress were noted in plane strain tests. From stress measurements it would appear that a von Mises failure criterion (maximum octahedral shear stress) can be used to relate unconfined failure conditions in triaxial and plane strain tests on this dense, saturated, frozen silt.

Acknowledgements

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