A frost heave interface condition for use in numerical modelling

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The object of this study was to develop a condition which, when applied at the interface between frozen and unfrozen soils in a numerical model of the ground thermal regime, would describe the relative rates of frost heave and frost front advance that would occur. The starting point for this calculation was a basic physical model of the frost heave mechanics. The result is in the form of a semi-empirical expression which is fitted to experimental data. This form is similar to some of the other empirical frost heave expressions that have been proposed. It thus provides a physical basis for them and gives some guidance as to their likely ranges of validity.

Le but de cette étude était d'obtenir une formule qui, lorsqu'appliquée à l'interface entre sols gelés et non gelés dans un modèle numérique du régime thermique du sol, décrirait les vitesses relatives du soulèvement attribuable au gel et de la progression du front de gel. Le point de départ de ces calculs était un modèle physique de base de la mécanique du soulèvement par le gel. Les résultats se présentent sous forme d'une équation semi-empirique qui est ajustée aux données expérimentales. Cette équation ressemble aux autres expressions empiriques proposées pour décrire le soulèvement par le gel et leur fournit ainsi une base physique tout en apportant des indications quant à leurs limites possibles de validité.

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Introduction

There are many approaches that can be taken in frost heave modelling. One can start with experimental data and generalize it by relating heave rate to frost penetration rate and/or heat fluxes (Horiguchi 1979; Konrad 1980; Konrad and Morgenstern 1980; Nixon *et al.* 1982; Outcalt 1980; Ueda and Penner 1978). This approach has the advantage that the resulting empirical equations can be used directly in existing ground thermal regime models to give predictions under field conditions. There may, however, be some concern about extrapolating the laboratory-developed equations to field conditions partly because they do not have a physical basis.

An alternative is to use thermodynamic arguments combined with equations of energy and mass flow to construct a detailed physical model of the heaving process (Gilpin 1980*a*; Guymon *et al.* 1980; Hopke 1980; Miller 1978; Takagi 1980; Sheppard *et al.* 1978). This approach has the advantage of providing insight into the mechanics of frost heave; however, it has the disadvantages that the resulting model is usually very cumbersome to use as a predictive technique and may require the knowledge of more properties of the freezing soil than can reasonably be known.

In this paper an attempt is made to relate these two approaches. A basic physical model of frost heave developed by the author (Gilpin 1980*a*) is used as the starting point and, by simplifying it and considering only a restricted range of conditions, a form that coincides with some of the empirical correlations is developed. This gives a physical foundation for the correlation and perhaps gives some guidance as to its range of applicability. In the paper the relationship between only one of the available physical models and two of the empirical equations (Konrad 1980; Nixon *et al.* 1982) is explored. There are many similarities and differences between these and other available models which have not been explored in this paper due to space limitations. The nomenclature used here is defined in the Appendix.

The Model

The schematic diagram (Figure 1) represents what one might expect the freezing zone to look like when a frost front is advancing slowly into the soil. Of critical importance in this situation is the frozen fringe and its behavior. This region of the soil is below the freezing point; however, there is still a significant water movement through the region in the time scale of interest. If ice lensing is occurring in the soil the frozen fringe would be bounded on its cold side, T_ℓ, by the 'active' ice lens — that is the one that is presently growing. The furthest advance of the pore ice into the solid will be taken as the warm side limit, T_f, of the frozen fringe. For most soils other than perhaps clays, T_f can be taken as essentially 0°C. In the final form of the model which will be developed here the frozen fringe will be considered as a plane of negligible thickness which separates the frozen and unfrozen soil; however, to develop the set of the equations for this layer, it will first be considered as having some real thickness, *a*.

Energy and Mass Balances

It will be assumed that all of the latent heat releases in the soil occur either at the isotherm T_f, where most

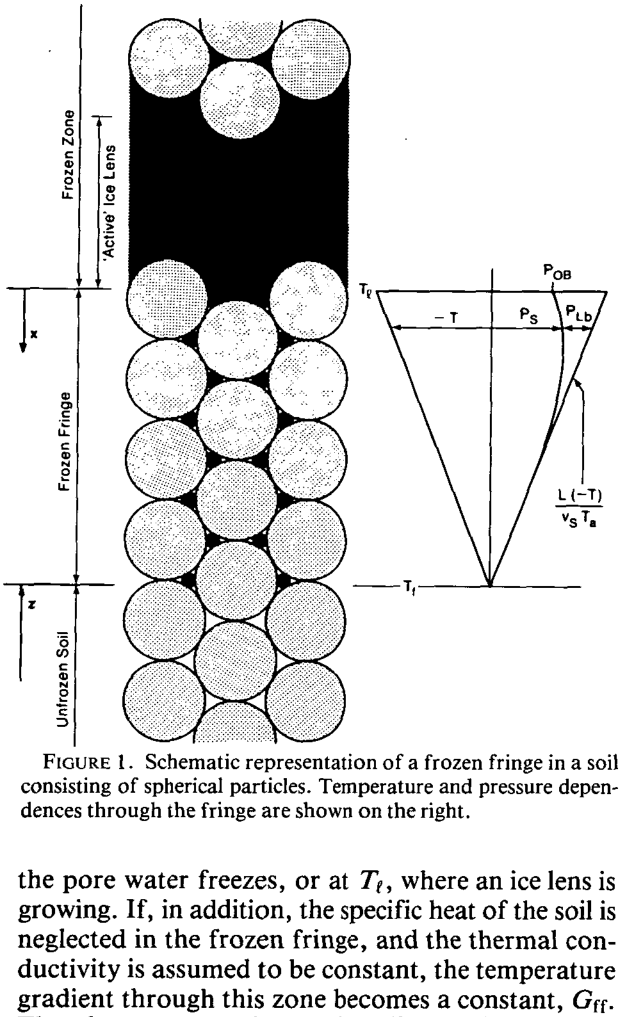

FIGURE 1. Schematic representation of a frozen fringe in a soil consisting of spherical particles. Temperature and pressure dependences through the fringe are shown on the right.

the pore water freezes, or at T_ℓ, where an ice lens is growing. If, in addition, the specific heat of the soil is neglected in the frozen fringe, and the thermal conductivity is assumed to be constant, the temperature gradient through this zone becomes a constant, G_{ff}. The above assumptions primarily require that the frost front is advancing slowly or that it is stationary. In most practical problems this will not be a bad assumption. With the temperature gradient in the frozen fringe constant, energy balances at the boundaries of the frozen fringe can simply be written:

$$[1] \quad k_f G_f - k_{ff} G_{ff} = \frac{L}{v_S} V_H \text{ at } T_\ell$$

and

$$[2] \quad k_{ff} G_{ff} - k_{uf} G_{uf} = \rho_{sio} L \frac{dz}{dt} \text{ at } T_f .$$

In equation 2, ρ_{sio} is the mass of ice that forms per unit volume of soil when the soil passes through T_f which will be assumed to be $\rho_{soil}\, w_c\, v_L/v_S$.

The heave rate, V_H, can be related the water fluxes in the frozen fringes, V_{ff}, and the unfrozen soil, V_{uf}, by considering the mass balances at T_ℓ and T_f. This gives

$$[3] \quad V_{ff} = \frac{v_L}{v_S} V_H$$

and

$$[4] \quad V_{uf} = V_{ff} + \rho_{sio} \frac{v_L}{v_S} (v_S - v_L) \frac{dz}{dt} .$$

The Heave Equation

An equation for the mass flux in the frozen fringe was obtained by the author earlier (Gilpin 1980*a*):

$$[5] \quad V_{ff} = K_f \frac{v_S}{g} \frac{d}{dx} \left[P_S + \frac{LT}{v_S T_a} \right]$$

where the mass flux is expressed as a mean water velocity, V_{ff}. Written in this form, equation 5 emphasizes the fact that a mass flow can occur from a region of low to a region of high ice pressure, P_S, if it is driven by a temperature gradient. The derivation of equation 5 assumes that thermodynamic equilibrium exists locally throughout the frozen fringe. Also it is assumed that over the temperature and pressure ranges of interest the Clausius-Clapeyron expression can be made linear and integrated to give

$$[6] \quad P_L = P_S + \frac{L}{v_S T_a} T$$

where P_L is the pore-water pressure. Also in this expression terms of order $\Delta v/v$ have been neglected compared to terms of order one. It is apparent from equation 6 that the driving force for mass flow in equation 5 could equivalently been taken as a gradient of P_L.

The boundary conditions appropriate to equation 5 will now be developed at T_f,

$$[7] \quad P_S + \frac{L}{v_S T_a} T_f = P_f$$

where P_f is the pore-water pressure just below the frost front. This pressure could be related back to the pressure of the water source using the hydraulic resistance of the unfrozen soil; however, in many cases the hydraulic resistance of the unfrozen soil can be neglected in comparison to that of the frozen fringes.

In the model developed by the author (Gilpin 1980*a*), the conditions at T_ℓ were found to fluctuate as the rhythmic ice banding proceeded. For the purpose of this model, in which the prediction of specific ice lenses is not required, it will suffice to use the 'mean' or 'typical' conditions referred to earlier (Gilpin 1980*a*). This would suggest that at T_ℓ one might take,

$$[8] \quad P_S = P_{OB} + P_o$$

and

[9] $dP_S/dx = 0.$

Equation 8 says that the pore-ice pressure at T_ℓ must be approximately equal to the overburden pressure, P_{OB}, plus an additional pressure, P_o, required to separate the soil particles. A physical approximation to P_o is

[10] $P_o = 4\sigma_{SL}/d_c$

where d_c is the diameter of the critical soil particle fraction (perhaps something like the finest ten per cent). Alternatively P_o can be taken as a soil property which can be estimated as shown later from frost heave data. The other part of the ice-lensing condition, equation 9, implies that, in the mean, P_S in the frozen fringe does not rise above the value given by equation 8. Using the condition in equation 9 along with equations 3 and 5 gives

[11] $$\frac{V_H}{G_{ff}} = \frac{v_s}{g\,v_L}\,\frac{L}{T_a}\,K_{f1}$$

where K_{f1} is the permeability of the partially frozen soil at the base of the active ice lens. Konrad (1980) has called the ratio V_H/G_{ff} the segregation potential, *SP*, and has used it to describe the frost-heave characteristics of a soil.

To express the segregation potential in terms of externally applied conditions such as overburden pressure, equation 5 must be integrated between T_f and T_ℓ and thus the dependence of K_f must be known. The author (1980*a*) argued that for a rigid soil matrix, that is one with low compressibility, K_f should be primarily a function of temperature and that a reasonable approximation to K_f was one of the form

[12] $K_f = K_1(-T)^{-\alpha}.$

With this expression the integration of equation 5 can be completed to give

[13] $$T_\ell = -\frac{v_S T_a}{L}\,\frac{\alpha+1}{\alpha}\,(P_{OB} + P_o - P_f).$$

Using this value of T_ℓ in equation 12 and putting the result in equation 11 gives an explicit equation for the segregation potential:

[14] $$SP = \frac{v_S}{g\,v_L}\,\frac{L}{T_a}\,K_1 \times \left[\frac{v_S T_a}{L}\,\frac{\alpha+1}{\alpha}\,(P_{OB} + P_o - P_f)\right]^{-\alpha}.$$

The properties of the soil in the frozen fringe that appear in this expression are P_1, P_o, and α. An additional property, the thermal conductivity in the frozen fringe, appears in the heat balance equations and can conveniently be included in a redefined segregation potential. In addition, the redefined segregation potential was made a dimensionless quantity, *DSP*, which represents the ratio of the latent heat release at the active ice lens to the heat flow through the frozen fringe,

[15] $$DSP = \frac{L\,V_H}{v_S\,k_{ff}\,G_{ff}}.$$

For practical purposes the dimensionless segregation potential, *DSP*, can then be expressed in a quasi-empirical form

[16] $DSP = C_1(P_{OB} + P_o - P_f)^{-\alpha}$

where C_1, P_o, and α are to be determined by fitting to experimental data. The above calculations do not imply that the form of equation 16 is the only form that may be used. For example, if instead of the power law dependence of equation 12, an exponential decrease of frozen fringe permeability with temperature below freezing was assumed, the resulting *DSP* would have taken the approximate form

[17] $DSP = C_2\exp[-(P_{OB} - P_f)/P_o]$

at least for $P_{OB} \geqslant P_o$. This form was used by Konrad (1980) to fit his segregation potential data. In this case the dependences can not be expressed in a simple analytical way for small overburden pressures. The forms in equations 16 and 17 will be fitted to experimental data in a later section.

The Frost Heave Interface Conditions

The previous section gives a physical foundation (at least for the case of a slowly advancing ice front) to the argument of Konrad (1980) that the segregation potential is a fundamental parameter related to known or calculated pressures. The segregation potential by itself, however, only relates the heave rate to an unknown, the heat flux in the frozen fringe. The energy balance conditions in equations 1 and 2 must be used with the *DSP* expression to relate heave rate to conditions outside the frozen fringe. The resulting expressions will be called the frost heave interface conditions.

It will be assumed that the frozen fringe is an interface surface separating the frozen and unfrozen soil. Also assumed is the existence of thermal models with which the heat fluxes or temperature gradients on either side of such an interface could be calculated. If the hydraulic permeability of the unfrozen soil is low it may also be necessary to use a soil water flow model to calculate P_f below the interface.

With the temperature gradients known the net heat flow, Q, out of the interface can be calculated

[18] $Q = k_f G_f - k_{uf} G_{uf}$.

Combining equations 1 and 2 gives an overall heat balance for the frozen fringe

[19] $$\rho_{sio} L \frac{dz}{dt} = Q - \frac{L}{v_S} V_H.$$

In the absence of frost heave, $V_H = 0$, equation 19 is the interface condition used in thermal modelling to calculate the advance of the frost front. The purpose of the present study is to incorporate the heave equation into the interface condition so that it will also apply when $V_H \neq 0$.

The interface condition will take different forms depending on whether the frost front is advancing, stationary, or retreating and each case must be considered separately.

A. *Advancing Frost Front*

Combining equations 16 and 1 gives

[20] $$V_H = \frac{v_S k_f G_f / L}{(P_{OB} + P_o - P_f)^a / C_1 + 1}.$$

Equations 19 and 20 then specify the rate of advance of the interface into the unfrozen soil, dz/dt, and the rate of generation of new frozen 'soil', V_H, on the cold side of the interface. It may also be of interest to calculate the soil ice content behind the advancing frost front. This would be

[21] $$\rho_{si} = (-\rho_{sio} \frac{dz}{dt} + V_H/v_S)/(-\frac{dz}{dt} + V_H).$$

B. *Stationary Frost Heave*

If equations 19 and 20 predict that $\frac{dz}{dt} \geqslant 0$, that is that a stationary or retreating frost front exists, the assumptions on which the segregation potential equation were derived are invalid and equation 20 can no longer be used to calculate V_H. If, however, Q is still positive, that is there is a net heat extraction from the frost heave, one can set $\frac{dz}{dt} = 0$ in equation 19 to get V_H from a simple energy balance

[22] $$V_H = \frac{v_s}{L} Q.$$

In this case the new 'soil' volume generated is pure ice:

[23] $\rho_{si} = 1/v_S$.

C. *Retreating Frost Heave*

If

[24] $Q < O$

that is there is a net heat flow into the frost front, thawing will occur. This in general results in a thaw consolidation problem (McRoberts *et al.* 1978); however, if the thawing occurs slowly enough that the soil is always fully consolidated locally we could simply write

[25] $$\rho_{si} L \frac{dz}{dt} = Q$$

for the interface condition.

Soil Parameters from Frost Heave Data

To obtain the *DSP* for a soil at a given overburden pressure the heave rate for a given temperature gradient or heat flux in the frozen fringe must be measured. Konrad (1980) has pointed out that one of the better ways to determine their segregation potential is in a one-dimensional freezing test where both end temperatures are fixed. In such a test the segregation potential can most easily be calculated just when the frost front advance ceases. The conditions required for equation 5 are still valid and, at that point, equation 2 can be used to relate G_{ff} to G_{uf} so that *DSP* can be determined using measurable temperature gradients,

[26] $$DSP = \frac{L V_H}{v_S k_{uf} G_{uf}}.$$

If the gradient G_{uf} is not known, G_f can be used in equation 1 to give

[27] $$DSP = \frac{1}{\dfrac{v_S k_f G_f}{L\, V_H} - 1}.$$

Ueda and Penner (1978) noted that the heave rate is fairly constant over a long time period at, and about, the time when the frost front progression ceases. This is particularily true of tests done at high overburden pressures when a very long time period of a constant heave rate is observed.

Some of the available frost heave data are used to see how the measured *DSP* would fit the form of equations 16 and 17. In Figures 2 and 3, data obtained from Ueda and Penner (1978) for five types of MacKenzie Valley silt and two types of Calgary silt are reduced to this form and plotted as a function of overburden pressure, P_{OB}. In these data, the values of the temperature gradients were not directly available but could only be inferred approximately from the cold side and the chamber temperatures. Also data on P_f are not available so it was assumed equal to atmospheric pressure. In addition, data from Konrad and

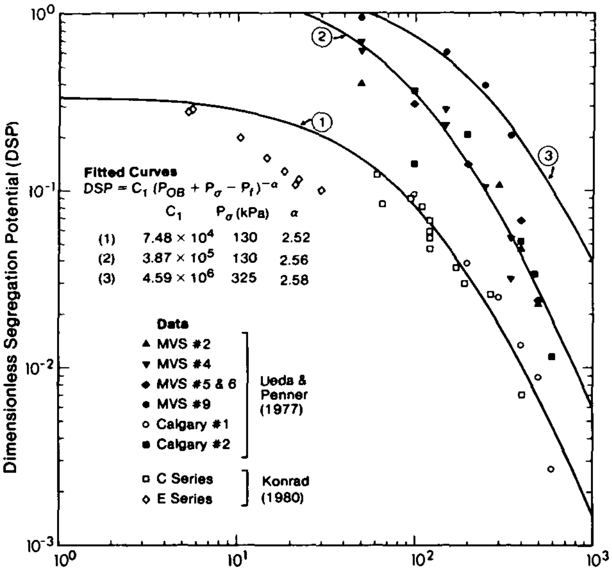

FIGURE 2. Experimental data fitted to a power law dependence of dimensionless segregation potential (*DSP*) on overburden pressure.

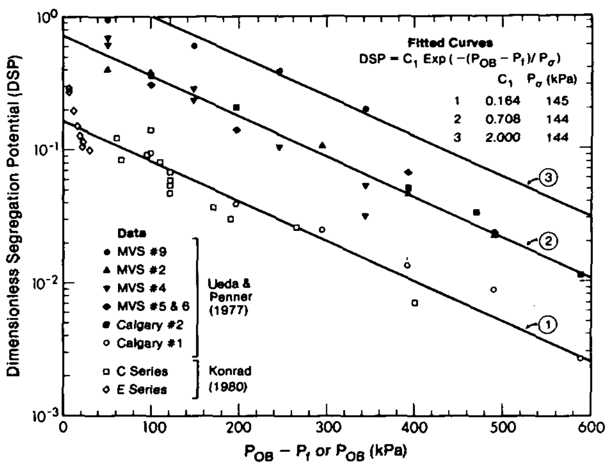

FIGURE 3. Experimental data fitted to an exponential law dependence of dimensionless segregation potential (*DSP*) on overburden pressure.

Morgenstern (1980) on Devon silt are plotted in the same form. For this data, temperature gradients were available and, for some cases (at low overburden pressure), the water suction at the frost front was also given. When the data were grouped into three groups of similar *DSP* results and were fitted by expressions of the form of equation 16 (*see* Figure 2) and equation 17 (*see* Figure 3), either form was found to fit the data satisfactorily. The correlation coefficients for equations 16 and 17 were 0.970 and 0.972 respectively, which indicates no clear preference for one or the other of the forms. The fitting parameters used are also shown on the figures. For an expression of the form of equation 16 a value of the power α around 2.5 appears satisfactory. This is close to the value of $\alpha = 2$ predicted theoretically by the author (Gilpin 1980*a*). The power law expression is also consistent with recent measurement of the variation in the thickness of the unfrozen film of water on a surface which indicates that over a wide range of temperatures this layer follows a power law dependence (Gilpin 1980*b*). For most of the data, the value of P_σ is in the range 130 to 145 KPa. In equation 10 this would correspond to a critical soil particle diameter of about 1μm. It is surprising that this value is so nearly constant. The only exception occurs for the soil MVS 9 for which the *DSP* is the highest and in fact appears to approach one asymptotically. The remaining constant, C_1 in equation 16 and C_2 in equation 17, is the only parameter that shows a strong dependence on soil type. This is convenient since it suggests that only a limited number of tests on any one soil may be required in order to adequately characterize the *DSP* of the soil over a wide range of overburden pressures.

At this point, it may be useful to relate the *DSP* to another parameter sometimes used to analyse frost-heave data — the incremental, ice segregation ratio (ΔISR). The ΔISR is the ratio of the rate of heave to the rate of thickening of the frozen zone (Nixon *et al.* 1981). Using the heat balance equation 1 along with the definition of the *DSP* gives

$$[28] \quad \Delta ISR = 1/\left(1 + \frac{\rho_i}{\rho_{sio}} \frac{Q_z}{DSP\,(Q_z + Q_{uf})}\right)$$

where

$$[29] \quad Q_z = \rho_{sio} L \frac{dz}{dt} \text{ and } Q_{uf} = k_{uf} G_{uf}.$$

When $Q_{uf} << Q_z$, the ΔISR can be directly related to the parameter *DSP* and both are dependent only on pressures. Nixon *et al.* (1981) found that during frost-heave tests there is a fairly long period of heave, which they call zone 2, where ΔISR is a constant. During the initial very high rate of frost penetration and also when the frost penetration ceases the ΔISR has different values. Equation 28 implies that ΔISR should go to one as Q_z; that is that the frost penetration rate goes to zero. This is consistent with the observations. Measurements of the ΔISR for intermediate frost penetration rates in Calgary silt give values in the range of 20 to 30 per cent for an overburden pressure of the order of 75 KPa. Assuming $Q_{uf} = 0$ in these tests gives values of $DSP = 0.80$ to 1.4 which, when plotted on Figure 2 or 3, would be consistent with the empirical curve #3 in these figures.

Limitations on the Applicability of the DSP

By examining the assumptions used in the derivation of the *DSP* and some of the experimental measurements related to it one can get some idea of the bounds beyond which it would not be applicable. One of the basic assumptions used was that the frost front was advancing slowly. This assumption led to the simplification of the latent heat release occurring either at the frost front or at the active ice lens and the soil in between being in a quasi-steady state. For rapid rates of cooling in the frozen fringe this simplification undoubtedly fails.

Konrad (1980) has measured the effect of cooling rate on the *SP* laboratory frost-heave tests. He finds that, for rates of cooling at the frost front greater than about 0.01°C/hr, the *SP* becomes a function of cooling rate as well as pressure. Also ΔISR measurements show that, at high rates of frost penetration (greater than about 10 mm/day), the value of this parameter is suppressed relative to its more constant value at slower penetration rates (Nixon *et al.* 1981). Because of the connections between these parameters indicated in the previous section the same consequence would be implied for the *DSP*. Fortunately many of the problems of practical interest would have slow frost penetration rates for which this limitation is not important.

The author's argument that led to the permeability in the frozen fringe being a function only of temperature relied on the assumption of a rigid soil particle matrix (Gilpin 1980*a*). However, observations, suggest that, particularily for a soil with high clay content, the ice pressures developed on freezing result in large micromorphological changes in the soil particle structure. These undoubtedly effect permeability. If in place of equation 12 a permeability that was a function of effective stress was assumed, the nature of the result for *DSP* would not be fundamentally altered. A more complex pressure dependence would, however, be implied.

Since the derivations in this paper apply to a saturated soil only two phases were assumed to be present — liquid and solid. However, because of the high suction pressures developed in the frozen fringe it is quite possible that a vapour phase could be produced. This becomes a particularly strong possibility when it is considered that dissolved gases in the water will be concentrated at the base of the active ice lens thus decreasing the suction that the pore water can withstand. The formation of a vapour phase would not change the basic transport equation 5; however, it may introduce a dependence of permeability or of P_a on the pore-water pressure. This in turn means that *DSP* may have a dependence on the water pressure, P_f, other than its appearance in the effective stress term $P_{OB} - P_f$. Konrad (1980) has in fact found that some of his data, in particular his Series E tests, in which P_f was varied without varying P_{OB}, show a stronger dependence on P_f than would occur if its only effect was through an effective stress.

In the present analysis the only water migration in the frozen soil that is accounted for is that which occurs in the frozen fringe. Water migration in soil colder than the active ice lens was observed by the author in the model (Gilpin 1980*a*). This migration was not important for short duration tests; however, for longer times, in particular geological times, it is undoubtedly important. Also, as Mackay and others have pointed out (Mackay *et al.* 1979), temperature gradients in field problems may be very small in which case the frozen fringe may become a zone several metres in thickness. Water migration and storage in a large zone of frozen soil is then important. When the frozen fringe is large, the accumulation of ice in the frozen zone due to expansion of the soil matrix may account for all the heave of the soil. The result may then be a quite different behavior in which no visible ice lenses occur.

Conclusion

The segregation potential as proposed by Konrad and Morgenstern (1980) or the *DSP* as described in this paper were shown to have a physical basis in the mechanics of frost heave. It is suggested, therefore, that it is a good starting point from which to develop a frost heave interface condition for use in numerical thermal regime modelling of a frost-susceptible soil. When tested against some of the available experimental data it appears that it correlates the data for various soils satisfactorily. Much more work remains to be done, however, to determine the range of conditions for which it is applicable. This work, which is proceeding, involves comparing predicted and measured heave rates for a wide range of different field and laboratory tests.

Acknowledgement

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Appendix

Nomenclature Defined

a – thickness of the frozen fringe
C_1, C_2 – fitting constants in DSP expressions
d_c – particle size diameter critical in determining heave behavior
DSP – dimensionless segregation potential
g – acceleration due to gravity
G_f, G_{ff}, G_{uf} – temperature gradients in the frozen, frost fringe, and unfrozen soils
k_f, k_{ff}, k_{uf} – thermal conductivities in the frozen, frost fringe, and unfrozen soils
L – Latent heat of fusion
P_S, P_L – pressures in the solid (ice) and liquid (water) phases
P_{OB} – overburden pressure
P_f – pore water pressure at the frost front
P_o – excess pressure (effective pressure) required for the formation of a new ice lens.
Q – net heat flux to frozen fringe
SP – segregation potential
t – time
T_ℓ – temperature at the base of the active ice lens
T_f – temperature at the frost front
T_a – absolute temperature (taken as 273 K)
v_S, v_L – specific volumes of solid (ice) and liquid (water) phases
V_H – total heave rate
V_{ff}, V_{uf} – water flow rates in the frozen fringe and the unfrozen soil
w_c – soil water content
x, z – dimensions as shown in Figure 1
α – power in empirical permeability equation
K_f, K_{f1}, K_{uf} – permeabilities in the frozen fringe, in the frozen fringe at temperature T_ℓ, and in the unfrozen soil
K_1 – constant in empirical permeability equation
ρ_i – density of ice
ρ_{sio}, ρ_{si} – density of ice in the soil produced by freezing of pore water and total ice density
σ_{SL} – ice–water interfacial tension