A MODEL FOR THE CLARIFICATION OF PERENNIAL GROUND ICE BY THERMALLY-INDUCED REGELATION

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Abstract

A model is presented for the clarification of perennial ground ice by thermally-induced regelation. This is a phenomenon whereby hygroscopic particles embedded in ice migrate up a temperature gradient. Velocities increase exponentially as the temperature approaches zero. In thermo-active permafrost, warm summer temperatures induce particle movement towards the active layer. In winter, the thermal gradients are reversed and transport occurs in the opposite direction. The average temperature for upward migration is warmer than that for downward migration. Thus, grains should experience a net upward displacement. Below the depth of zero annual amplitude, downward particle migration takes place in response to the geothermal gradient. Equations are formulated for both processes assuming a fixed ice lattice, silt-sized grains and depth positive downward. The theory of non-equilibrium thermodynamics is used to describe the volume flux of particles. An empirical equation is derived for the coupled transport coefficient. Combining these expressions with analytical solutions for the temperature and thermal gradient yields flux equations in terms of depth and time. Maximum fluxes of \(-7.8 \times 10^{-11}\) and \(+1.3 \times 10^{-11}\) m.s\(^{-1}\) were calculated for the annual temperature cycle and geothermal gradient, respectively. Over geological time, the model predicts ice-enrichment of the coldest permafrost, which is compatible with the observed vertical distribution of massive ground ice bodies.

Introduction

Ice-enriched zones are frequently encountered in permafrost. In extreme cases, ice-enrichment may approach 100% by volume and extend for many meters, both horizontally and vertically (Pollard and French, 1980). Under such conditions, the ice is referred to as massive ground ice. Data from drilling operations provide an indication of the vertical variability of ground ice bodies. Massive ice may be found anywhere from the base of the active layer to depths of 40 m or more (Rampton and Mackay, 1971). However, the highest ice contents tend to occur close to the permafrost table (Outcalt, 1982).

The genesis of massive ground ice is not always clear. Some bodies can be attributed to the burial of glacier ice and/or compacted snow banks (Mackay, 1971). Of more interest, however, is the origin of what has been termed "epigenetic" massive ice. Mackay (1971) proposed that this ice is the result of long-term ice segregation, associated with
climatic cooling. In theory, excess water is supplied under high pressure by the closed system freezing of underlying sands. A drawback to this hypothesis is the apparent discrepancy between rates of climatic change and rates of heat extraction necessary to maintain ice segregation beneath an increasing overburden.

It is also possible that ice segregation may occur as a result of thermally-induced migration of ground water through the permafrost (Harlan, 1974). However, because of the low hydraulic conductivities for frozen porous media, the rate of upward migration is much too small to account for the high ice contents observed just below the active layer. Furthermore, there is some controversy as to whether ice segregation can occur within already frozen materials (Konrad, 1986; Williams and Wood, 1986).

Recently, it has been suggested that epigenetic ground ice may be the result of annual additions of water from the atmosphere (Cheng, 1983; Burn and Smith, 1985). In summer, water from snowmelt and precipitation flows down through the thawed active layer in response to gravity and/or thermally-induced gradients of water potential (Mackay, 1983; Burn and Michel, 1988). In winter, the thermally-induced gradients are reversed and water tends to migrate upwards through the frozen active layer (Mackay et al., 1979; Harris, 1988). However, this migration is much slower due to the low hydraulic conductivity of frozen soil compared to unfrozen soil (Burt and Williams, 1976; Horiguchi and Miller, 1983). Furthermore, the atmospheric driving forces for water loss are relatively low under frozen conditions (Smith and Burn, 1987; de Jong and Kachanoski, 1988). Thus, a proportion of the annual precipitation will be trapped as ice at the base of the active layer due to the "law of unequal migration of unfrozen water" (Cheng, 1983). Water balance studies in permafrost regions indicate that year-to-year changes in ground ice storage may account for up to 10% of the annual atmospheric precipitation (Woo, 1986).

In addition to the above mechanism, Cheng (1983) has proposed that ice-rich permafrost may purify itself of mineral particles by thermally-induced regelation. This is a process whereby hygroscopic grains embedded in ice migrate up a temperature gradient, in much the same way that isolated pockets of brine migrate through sea ice (Hoekstra et al., 1965; Harrison, 1965). Experimental studies of the phenomenon have been conducted by Hoekstra and Miller (1967) and Römkens and Miller (1973). Their data show that particle velocities increase exponentially as the temperature approaches zero. A theoretical analysis of regelation around a single sphere has been presented by Philip (1980). Miller (1983) provides a qualitative discussion of thermally-induced regelation as it applies to frozen soil (i.e. a fixed particle matrix with respect to the ice).

In the case of massive ground ice (i.e. a fixed ice lattice with respect to the mineral grains), particle migration may occur in response to surficial temperature oscillations and/or the geothermal gradient (fig. 1). In the thermo-active zone, warm summer temperatures induce transport of soil particles towards the base of the active layer. In winter, the thermal gradients are reversed and movement occurs towards the depth of zero annual amplitude. On average, the temperature at which upward migration takes place will be warmer than the temperature for downward migration. Thus, particles within the thermo-active zone should experience a net upward displacement with respect to a fixed ice lattice (Cheng, 1983). Below the depth of zero annual amplitude (i.e. the thermo-inactive zone) further purification may occur as particles migrate downward under the influence of the geothermal gradient.

A significant feature of the above scenario is that ice segregation appears to take place within a closed system. Assuming an initially super-saturated homogeneous mixture, thermally-induced regelation will result in a redistribution of mineral grains relative to the ice lattice. Over geological time, the embedded particles can be expected to congregate at the warm end of the system, while ice will tend to occupy the cold end as a pure phase. In this context, it is unnecessary to invoke a continuous external water source to explain the ice-enrichment of permafrost.

This paper presents a model for the clarification of perennial ground ice by thermally-induced regelation. Its major objective is the derivation of equations describing the migration of embedded mineral particles in terms of depth and time. These equations will then be used to compare the relative magnitudes of the particle fluxes induced by annual temperature oscillations at the ground surface and the geothermal gradient, respectively. The model's predictions will also be discussed in relation to published estimates of the vertical extent and age of massive ground ice bodies.
Flux Equation

Transport phenomena within frozen non-heaving porous media can be described using the theory of non-equilibrium thermodynamics (Perfect et al., 1989). Applying this theory, we can write the following expression for the total flux of water within a liquid-ice-mineral matrix in response to gradients of ice pressure ($\nabla P_i$), temperature ($\nabla T$), osmotic potential ($\nabla \Pi$) and electrical potential ($\nabla \psi$):

$$j_w = -L_w \nabla P_i - L_T \nabla T - L_\Pi \nabla \Pi - L_\psi \nabla \psi$$  \hspace{1cm} [1]

where $(j_w)_i$ is the volume flux of water expressed in terms of the ice phase, $L_w$ is the straight (conjugated) transport coefficient, and $L_T$, $L_\Pi$ and $L_\psi$ are cross (coupled) transport coefficients.

Assuming $\nabla P_i = 0$, $\nabla \Pi = 0$ and $\nabla \psi = 0$ eqn. [1] simplifies to:

$$(j_w)_i = -L_T \nabla T$$  \hspace{1cm} [2]

In an ice-rich mixture (i.e. individual particles not constrained by the presence of neighboring particles) the volumetric flux of mineral material is related to the volumetric flux of water at the Darcy scale. For a closed system this relationship can be written as:

$$j_s = -(j_w)_i$$  \hspace{1cm} [3]

where $j_s$ is the volume flux of mineral particles.

Using eqn. [2] in eqn. [3] produces the following expression for the flux of mineral material:

$$j_s = L_T \nabla T$$  \hspace{1cm} [4]

Römkens and Miller (1973) measured the migration of silt-sized particles embedded in ice, in response to various temperature gradients. The velocity of each particle increased as it approached the 0°C isotherm. However, the rate of movement was not systematically related to the temperature gradient over the range 75 to 174 °C.m⁻¹. For the purposes of this exercise, it will be assumed that the mineral flux is a linear function of the driving force (temperature gradient).

Transport Coefficient

The transport coefficient in eqn. [4] will be temperature dependent. Assuming $j_s$ is proportional to the temperature gradient, the data of Römkens and Miller (1973) suggest an exponential relationship between $L_T$ and temperature below zero. Equating their velocity with our flux (i.e. no interaction between mineral grains at the Darcy scale) this function can be modeled as follows:

$$j_s / \nabla T = L_T / T = ab^{nT}$$  \hspace{1cm} [5]

where $a$ and $b$ are constants.

Least squares estimates of $a$ and $b$ for a mineral particle with a radius of $1.27 \times 10^{-5}$ m exposed to a temperature gradient of 114 °C.m⁻¹ (fig. 4 in Römkens and Miller, 1973) are $2.89 \times 10^{-10}$ m².s⁻¹°C⁻¹ and 12.75 °C⁻¹ ($n = 5$, $R^2 = 0.994$, $p < 0.01$), respectively.

The transport coefficient will also change as a function of particle size. However, no quantitative data are available on the nature of this relationship. In the absence of such data, eqn. [5] can only be used if one assumes the mineral fraction is made up exclusively of silt-sized particles with a mean radius of $1.27 \times 10^{-5}$ m.

Expressions for $j_s$

THERMO-ACTIVE ZONE

Expressions for the temperature ($T$) and thermal gradient ($\nabla T$) in eqn. [5] can be obtained by solving the problem of heat conduction without phase change for a homogeneous, infinite ice-rich body of permafrost subject to sinusoidal annual temperature fluctuations at the top of the thermo-active zone. It should be noted that this scenario may represent a considerable oversimplification of reality. Actual temperature fluctuations at the base of the active layer can be expected to depart from a perfect sine wave due to the "zero-curtain effect" (Washburn, 1979). Furthermore, the effect of the geothermal gradient is not taken into account within the fluctuating zone; uncertainties in the approximations of the sinusoidal model are greater than the effect of deep earth heat flow.

A formal statement of the above problem can be found in Carslaw and Jaeger (1959) and Lunardini (1981). The system of equations is solved by separation of variables. The solution given on p. 143 of Lunardini (1981) for $t \gg 0$ can be written as follows:

$$T = T_0 + T_a e^{-\alpha} \cos (2\pi t / p - rz)$$  \hspace{1cm} [6]

where $T_0$ is the mean annual temperature at the base of the active layer, $T_a$ is the amplitude of the temperature oscillation at the base of the active layer, $t$ is time, $r$ is the attenuation coefficient = $\sqrt{\pi / \alpha p}$, $p$ is the period of oscillation, $\alpha$ is the thermal diffusivity = $1.15 \times 10^{-6}$ m².s⁻¹ (the value for pure ice given in Carslaw and Jaeger, 1959), and $z$ is depth, positive downward.

Assuming the temperature oscillation at the base of the active layer peaks at 0°C, then $T_a = -T_0$. Differentiating eqn. [6] with respect to $z$, we obtain the following expression for the temperature gradient as a function of depth and time:

$$dT / dz = rT_a e^{-\alpha} \{ \sin (2\pi t / p - rz) - \cos (2\pi t / p - rz) \}$$  \hspace{1cm} [7]

Substituting eqn.'s, [6] and [7] into eqn. [5] we obtain the following expression for the flux of solid material within the thermo-active zone:

$$j_s = arT_a \{ \sin (2\pi t / p - rz) - \cos (2\pi t / p - rz) \} x e^{-\alpha} + b \{ T_0 + T_a e^{-\alpha} \cos (2\pi t / p - rz) \}$$  \hspace{1cm} [8]

Eqn. [8] has been used to plot $j_s$ as a function of time for a single temperature oscillation (fig. 2). It can be shown that $j_s = 0$ at $t / p = 0.125$ and 0.625. These discontinuities, represented by the solid vertical lines, separate the upward
Figure 2. The flux of mineral material ($j_s$) versus time for a complete temperature oscillation, assuming $z = 0$, $T_a = -T_0 = 10 \, ^{\circ}C$, $a = 2.89 \times 10^{-10} \, m^2.s^{-1}.^{\circ}C^{-1}$, $b = 12.75 \, ^{\circ}C^{-1}$ and $\alpha = 1.15 \times 10^{-6} \, m^2.s^{-1}$ in eqn. [8]. Arrows indicate the direction and relative magnitude of particle migration.

and downward fluxes. There is a tremendous amount of variation in $j_s$ (both positive and negative) over the period of the cycle. For most of the period, however, the flux of mineral material can be considered effectively zero; only 6% of the time (i.e. approximately 3 weeks assuming $p = 1$ yr.) is $j_s > 10^{-10} \, m.s^{-1}$. This range falls entirely within the upward displacement sector (fig. 2).

Table I presents sample calculations of $j_s$ for various combinations of temperature amplitude ($T_a$) and depth below the active layer ($z$), assuming the same input parameters as in fig. 2. Maximum positive values of $j_s$ range from $10^{-52}$ to $10^{-20} \, m.s^{-1}$, which for all practical purposes constitutes a zero downward flux. In contrast, maximum negative fluxes range from $-10^{-33}$ to $-10^{-9} \, m.s^{-1}$. Clearly, net particle displacement will be upward and will be controlled by the maximum negative particle flux, $(j_s)_{max}$. This parameter becomes more sensitive to the amplitude of the temperature oscillation with depth (Table I).

Assuming $t \to p$ and $z \to 0$, eqn. [8] can be simplified to give the following expression for the maximum negative particle flux immediately below the active layer:

$$(j_s)_{max} = -arT_s$$  \[9\]

Assigning values of 5 and $15^{\circ}C$ to $T_a$, with the same input parameters as in fig. 2, eqn. [9] predicts a range of $(j_s)_{max}$ from $-4.4 \times 10^{-10}$ to $-1.3 \times 10^{-9} \, m.s^{-1}$. However, maximal upward fluxes only occur for a limited time each year as $t \to p$. Assuming this constitutes 6% of the period, the adjusted range for $(j_s)_{max}$ is $-2.6 \times 10^{-11}$ to $-7.8 \times 10^{-11} \, m.s^{-1}$, respectively. Over geological time, one would expect embedded mineral particles to be expelled into the active layer leaving ice as a pure phase in the thermo-active zone. This scenario is in agreement with field observations of the vertical distribution of massive ground ice (e.g. Rampton and Mackay, 1971).

**Thermo-Inactive Zone**

At lower depths we are not concerned with oscillating temperatures. Consequently, the only driving force for particle migration in the thermo-inactive zone is the geothermal gradient (fig. 1). The temperature increase associated with the geothermal gradient can be expressed in terms of depth using a simple linear model:

$$T = T_0 + cz$$  \[10\]

where $c$ is a constant.

Differentiating eqn. [10] with respect to depth we obtain the following expression for the geothermal gradient:

$$dT/dz = c$$  \[11\]

Now, using eqn’s. [10] and [11] in eqn. [5] we obtain the following expression for the flux of mineral material within the thermo-inactive zone:

$$j_s = ac \, e^{b \{T_0 + cz\}}$$  \[12\]

Because of the exponential decrease in $L_T/T$ with temperature, the maximum downward flux of particles due to

<table>
<thead>
<tr>
<th>Depth (z) (m)</th>
<th>Amplitude ($T_a$) (°C)</th>
<th>Range of fluxes m.s⁻¹</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Negative, $(j_s)$</td>
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<tr>
<td>0.0</td>
<td>5</td>
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</tr>
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<td></td>
<td>10</td>
<td>$\leq 10^{-10}$</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$\leq 10^{-09}$</td>
</tr>
<tr>
<td>0.1</td>
<td>5</td>
<td>$\leq 10^{-11}$</td>
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<tr>
<td></td>
<td>10</td>
<td>$\leq 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$\leq 10^{-12}$</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>10</td>
<td>$\leq 10^{-25}$</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$\leq 10^{-33}$</td>
</tr>
</tbody>
</table>

$\alpha = 1.15 \times 10^{-6} \, m^2.s^{-1}$, $a = 2.89 \times 10^{-10} \, m^2.s^{-1}.^{\circ}C^{-1}$ and $b = 12.75^{\circ}C^{-1}$
the geothermal gradient, \( (\pm j)_{\text{max}} \), can be expected to occur close to the permafrost base. The bottom of the permafrost is given by:

\[
z = -T_d/c = T_d/c \quad (13)
\]

Therefore, as \( z \rightarrow T_d/c \) eqn. [12] simplifies to:

\[
(\pm j)_{\text{max}} = ac \quad (14)
\]

Measured values of \( c \) in permafrost range from 0.017 to 0.045 OC.m\(^{-1}\) (Washburn, 1979). Using these values in eqn. [14] along with the estimate of \( a \) calculated previously, we obtain a range for \( (\pm j)_{\text{max}} \) of between 4.9\( \times 10^{-12} \) and 1.3\( \times 10^{-11} \) m.s\(^{-1}\).

Assuming complete redistribution over time, the mineral material should congregate as an indurated horizon of high strength and bulk density at the bottom of the permafrost, leaving pure ice above. Model validation is hindered by the lack of deep (i.e. > 35 m) drill hole data. One third of the shot logs examined by Mackay (1971) failed to reach the bottom of the massive ice. Thus, the nature of the basal material remains speculative.

**MODEL DEVELOPMENT**

An expression for the volume of mineral particles per unit volume bulk at any time and depth can be obtained by applying the principle of mass conservation and integrating over time. The divergence of flux equation is written as follows:

\[
dS/dt = \nabla \cdot j_s \quad (15)
\]

where \( S \) is the volume of mineral particles per unit volume of bulk.

Substituting eqn. [8] into eqn. [15] and differentiating the RHS with respect to \( z \) we obtain the following equation for \( dS/dt \) in terms of the annual temperature cycle:

\[
dS/dt = -ar^2 T_s e^{bT_0} \left( bT_s e^{aT} \left( 1 - \sin(2m) \right) \right) + \sin(\pi) \left( b e^{bT_s} \cos(2m) - rz + 1 \right) e^{aT} \left( b e^{bT_s} \cos(2m) - rz - 1 \right) \quad (16)
\]

where \( m = (2\pi t/p - rz) \).

The next step is to integrate eqn. [16] with respect to time, which will give an expression for \( S \) at any depth and time. This integration could not be completed in time for press. A promising strategy involves the use of a series expansion for \( e \) in eqn. [16]. Further work is required before a solution is obtained based upon surficial temperature oscillations.

In contrast, it is relatively easy to derive an expression for \( S \) in terms of the geothermal gradient. Substituting eqn. [12] into our equation for the conservation of mass (eqn. [15]) and differentiating the RHS with respect to \( z \) we obtain:

\[
dS/dt = -abc^2 e^b \left( T_0 + cz \right) \quad (17)
\]

Integrating eqn. [17] from \( t = 0 \) to \( t = t \) yields the following expression:

\[
S = S_0 - abc^2 e^b \left( T_0 + cz \right) \quad (18)
\]

where \( S_0 \) is the volume fraction of mineral material at \( t = 0 \).

Rearranging eqn. [18], we obtain an equation for the time taken to achieve complete thermal clarification, \( t_{s \rightarrow 0} \), as a function of depth within the thermo-inactive zone:

\[
t_{s \rightarrow 0} = S_0 / abc^2 e^b \left( T_0 + cz \right) \quad (19)
\]

Evaluating eqn. [19] for various combinations of \( S_0 \) and \( c \) as \( z \rightarrow T_d/c \) (using the same values of \( a \) and \( b \) as in fig. 2) yields a range of times from 1,332 to 16,172 yrs. (Table II). The time taken for \( S \rightarrow 0 \) increases exponentially with distance above the permafrost base. Thus, the predictions in Table II must be regarded as the absolute minima required for complete thermal clarification. As such, they are in qualitative agreement with published estimates of the age of massive near-surface ice beds (e.g. Mackay, 1971; Burn et al., 1986).

**Conclusions**

The model predicts downward migration of mineral particles in the thermo-inactive permafrost due to the geothermal gradient and net upward movement in the thermo-active permafrost under the influence of surficial temperature oscillations. Upward particle displacement occurs over a relatively narrow time band within each annual cycle. This movement becomes more sensitive to the amplitude of the temperature oscillation with depth. Annual temperature fluctuations appear to induce slightly faster clarification than the geothermal gradient. Over geological time, embedded particles will be either expelled into the overlying active layer or congregate as an indurated horizon of high strength and bulk density towards the base of the permafrost. Ice will tend to occupy the coldest permafrost as a pure phase, which is compatible with field observations of the vertical distribution of massive ground ice. Applying the principle of conservation of mass and integrating over time

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**Table II. Sensitivity analysis for \( t_{s \rightarrow 0} \) predicted by eqn. [19] assuming \( z \rightarrow T_d/c \).**

<table>
<thead>
<tr>
<th>( S_0 ) [-]</th>
<th>( c(\text{OC.m}^{-1}) = 0.02 )</th>
<th>( t_{s \rightarrow 0} ) 0.03</th>
<th>( t_{s \rightarrow 0} ) 0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.7 ( \times 10^{11} )</td>
<td>7.5 ( \times 10^{10} )</td>
<td>4.2 ( \times 10^{10} )</td>
</tr>
<tr>
<td>0.50</td>
<td>3.4 ( \times 10^{11} )</td>
<td>1.5 ( \times 10^{11} )</td>
<td>8.5 ( \times 10^{10} )</td>
</tr>
<tr>
<td>0.75</td>
<td>5.1 ( \times 10^{11} )</td>
<td>2.3 ( \times 10^{11} )</td>
<td>1.3 ( \times 10^{11} )</td>
</tr>
</tbody>
</table>

\( a = 2.89 \times 10^{-10} \text{m}^2\text{s}^{-1}\text{OC}^{-1} \) and \( b = 12.75 \text{OC}^{-1} \)

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yields an expression for the volume of particles per unit volume bulk at any time and depth within the thermoinactive zone. Values of the time taken for this parameter to approach zero are compatible with published estimates of the age of massive near-surface ground ice beds. Further research is required to derive a similar expression in terms of annual temperature oscillations.

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References


