IV. 4. SIMPLE GRAPHICAL METHODS FOR ESTIMATING THE LOCATION OF PERMAFROST UNDER SHALLOW LAKES AND RIVERS

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The investigations reported by Johnston and Brown (2) and Mackay (4) are concerned with the location of permafrost in the neighbourhood of shallow bodies of water. Because problems of this sort will probably occur with increasing frequency in the future, an introduction to some simple graphical methods for estimating the temperature regime in the ground and the resulting position of the permafrost table is presented here.

Wherever there is a body of water, whose mean annual temperature is above freezing, a thawing back of the permafrost under and about it will occur. The location of the permafrost table then depends on a number of factors, the principal ones being the mean annual water and ground surface temperatures and the geothermal gradient for the region. Other governing conditions are the size and shape of the bodies of water and the constancy of their shape and size with the passage of time. Obviously possession of detailed information on temporal variations will not probably be available, but because these would likely be quite gradual it is reasonable to assume steady-state conditions for the purpose of obtaining a general picture of the thermal regime in the ground.

The graphical method outlined in this paper relies on the fact that the resultant temperature in the ground can be considered as the sum of three separate temperature effects, these being:

(1) that resulting from a temperature difference between the water and the surrounding ground surface;
(2) the amount by which the temperature is higher than at the surface due to the geothermal gradient;
(3) the actual temperature of the ground surface.

The first of these temperature effects is depicted in Figure 1 where the temperatures near the straight edge of a large shallow body of water are shown for the case where there is no geothermal gradient. It will be readily recognized from symmetry that heat flows from the body of water to the ground in circular paths, while the isotherms, or lines of constant temperature, form radii emanating from the edge. By way of example, with the given values of lake and ground surface temperatures, the temperature directly under the edge would be equal to the average of the water and surface temperature values. If the ground surface temperature T_1 is simply subtracted at all points and

$T_2 - T_1$ set to equal ν_o, the simple schematic temperature distribution of Figure 2a is obtained. The body of water is considered to be at a temperature ν_o higher than the ground surface.

The geothermal gradient can now be added to the temperatures in Figure 2a to produce Figure 2b. Here the lines parallel to the ground surface are isotherms due to the geothermal gradient alone - i.e. in the absence of the body of water. By simply joining points of intersection of the geothermal isotherms and the radial isotherms from Figure 2a which have the same sum, it is possible to obtain the temperature profiles for the combined case. To complete the problem, the value of T_1 (the actual ground surface temperature at all points) is added. For example, if ν_o were 10°F and T_1 were 22°F then the 1.0 ν_o isotherm would outline the 32°F isotherm and the position of the permafrost.

In Figure 2c the same direct addition procedure is used to obtain the temperature regime under a river with parallel sides, in this case in the absence of a geothermal gradient. The procedure, in effect, is to superimpose a reversed copy of Figure 2a on itself separated by a distance 2d. When the temperatures due to the two sets of isotherms are added, the new isotherms are circles. Also, the temperature at the external ground surface becomes ν_o while that of the river becomes $2\nu_o$. By subtracting the value ν_o at all points, the circular isotherms in Figure 2d are obtained, to which the geothermal gradient effect can be added in the same manner as was done in Figure 2b. The curves in Figure 2d are the final resulting isotherms under a river.

A numerical example will illustrate this procedure using again a value of 10°F for ν_o and 2°F per 100 feet for the geothermal gradient. This means that the temperature regime for the 1.0 ν_o value of the geothermal gradient occurring at a depth of 500 feet, which in this case is also the width of the river has been constructed. If now the water temperature is 32°F, the ground surface is at 22°F and the position of the permafrost boundary is represented by the 1.0 ν_o curve. Were the water at 34°F, with the ground surface at 24°F, the 0.8 ν_o isotherm would represent the permafrost face, and it can be seen that in this case there is no permafrost under the central portion of the river.

From these last considerations, it is clear how critical the actual water and ground temperatures are to determine the location of the permafrost. It is necessary to know these temperatures with considerable accuracy in order to predict the permafrost position.

The examples of the edge of a large body of water and of a river are very useful in obtaining a rough idea of the distribution of permafrost in a given problem, but the majority of water bodies do not conform to such a simple geometry.*

For irregular areas the temperature is determined by a different method as depicted in Figure 3. Here the water area or areas are subdivided into circular sectors of radius R and angle θ. For these circular sectors a simple equation developed by Lachenbruch (3), is available for the temperature at any depth z under the apex. The sum of temperatures due to all sectors gives the temperature which would occur in the absence of a geothermal gradient, and to this can be added the temperature due to the geothermal gradient for that depth. For further details of this procedure the reader is referred to references (1) and (3). To reduce the labour involved in summation of this kind, the author has devised a programme for the Bendix G-15 computer which is now available to the interested reader.

In summary, it is obviously a simple matter to estimate the temperature in the ground under shallow bodies of water and thereby to estimate the permafrost location. The gross assumption of steady-state conditions will not always be warranted but will probably yield an acceptable result for many practical purposes.

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* The graphical method for the edge of a large body of water and a river is identical with that used by Werenskiold for estimating the permafrost location in the neighbourhood of glaciers and fjords (5).

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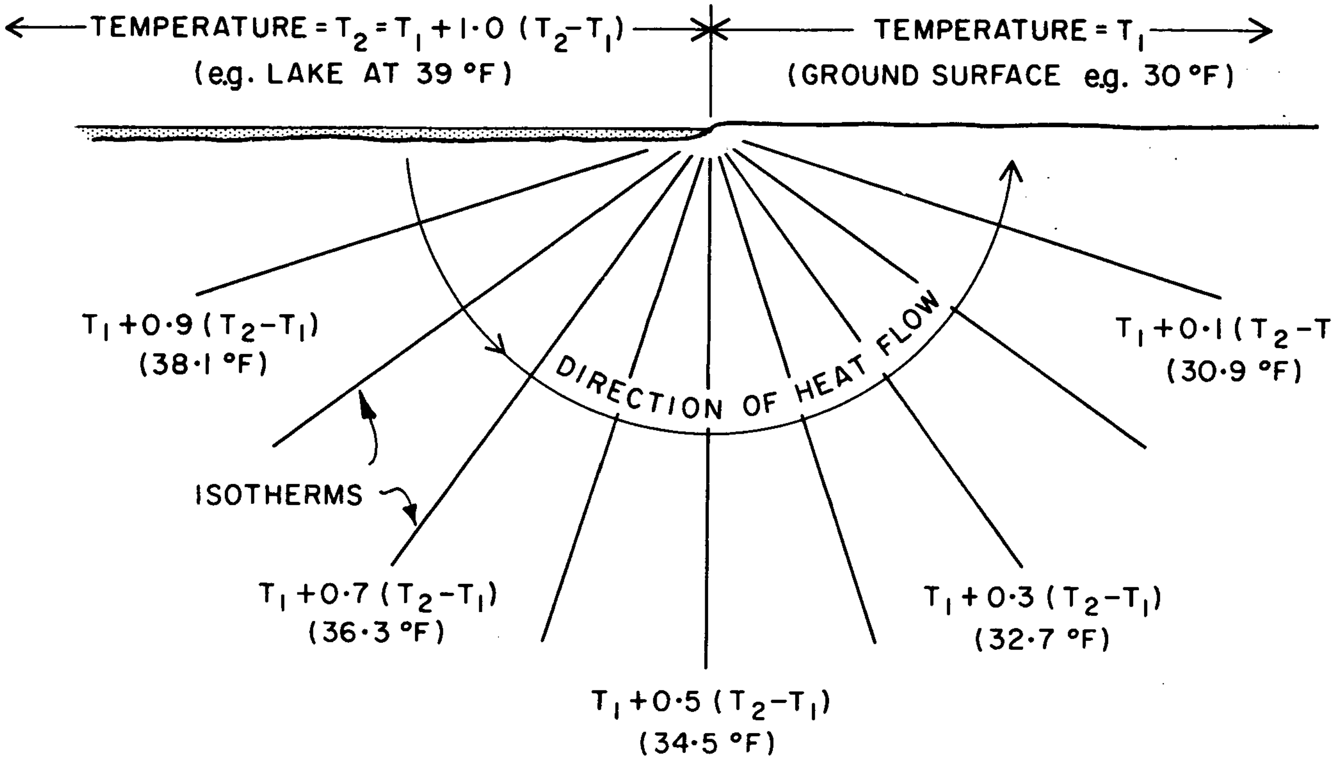

FIGURE I
STEADY TEMPERATURE UNDER THE STRAIGHT SIDE OF A LARGE, SHALLOW BODY OF WATER ON THE GROUND SURFACE

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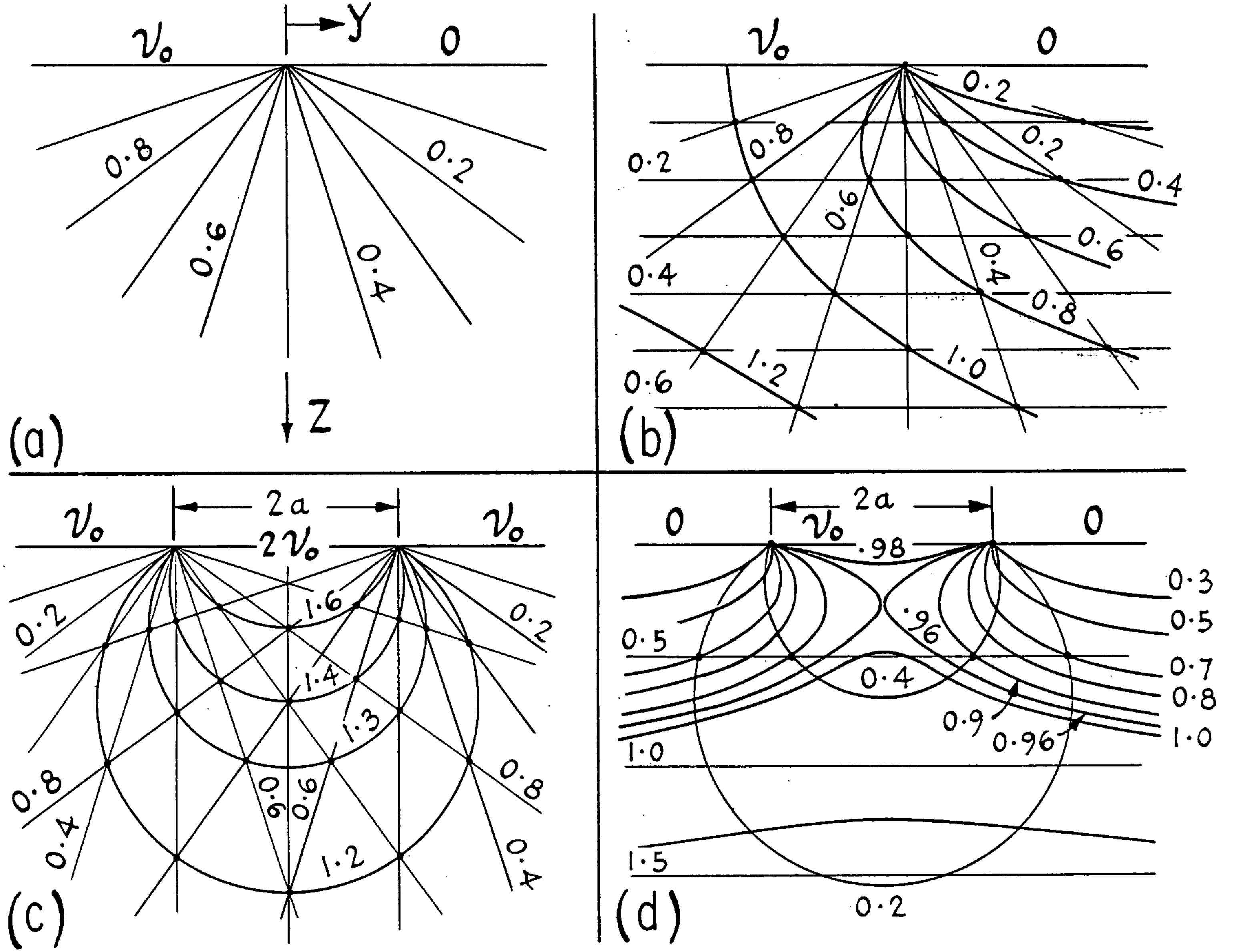

FIGURE 2

GRAPHICAL SUPERPOSITION OF TEMPERATURES

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$$\nu = \sum \frac{\theta}{360} \nu_0 \left[1 - \frac{1}{\sqrt{1 + (R/z)^2}} \right]$$

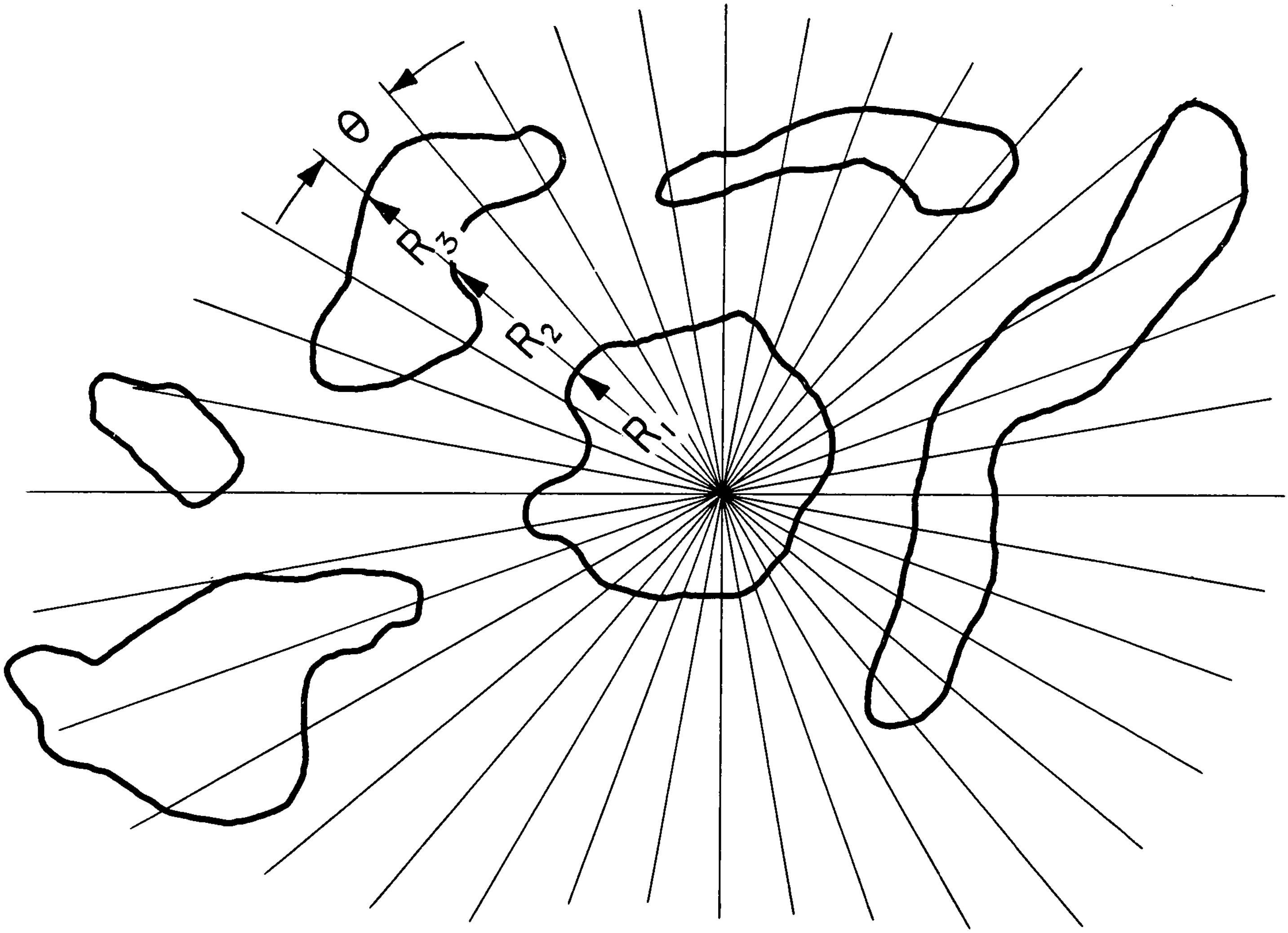

FIGURE 3
SUBDIVISION OF A LAKE-RIVER SYSTEM INTO CIRCULAR SECTORS FOR DETERMINATION OF TEMPERATURE UNDER THE COMMON APPEX