Strain dependence of Poisson's ratio for frozen sand

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Material properties of frozen soils are normally determined from unconfined and triaxial compression tests on cylindrical samples. Development of advanced analytical techniques, appropriate for design of many frozen soil structures, requires that data obtained from such tests be extended to provide constitutive relationships for more complex stress states. As part of a laboratory study on the mechanical properties of frozen silica sand, volumetric strain and Poisson's ratio were determined from uniaxial constant strain-rate and constant stress (creep) compression tests. Indirect tensile tests provided values for Poisson's ratio for biaxial stress conditions. The experimental data obtained from uniaxial constant strain-rate compression tests show that Poisson's ratio appears to be influenced by strain, strain rate, and temperature. Volumetric creep curves obtained from uniaxial, constant stress compression tests permitted development of an expression relating volumetric creep strain to applied stress. Poisson's ratio was observed to be relatively independent of applied compression stress level during creep and may be expressed as a power function of axial strain. For multiaxial stress states in which tensile and compression stresses are developed simultaneously on the principal axes, the results of indirect tensile tests suggest that a form of anisotropy may exist in the frozen sand as a result of different material behavior in tension and compression.

Les propriétés matérielles des gélisols sont habituellement déterminées au moyen d'essais de compression non confinée et triaxiaux, conduits sur des échantillons cylindriques. La mise au point de procédés analytiques avancés, convenant à la conception de nombreuses structures établies sur un gélisol, exige que l'on établisse des relations appropriées pour les états de contrainte plus complexes. Dans le cadre de l'étude en laboratoire des propriétés mécaniques d'un sable siliceux gelé, ont été déterminés la déformation volumétrique et le coefficient de Poisson au moyen d'essais de compression uniaxiale à vitesse de déformation constante et sous contrainte constante (fluage). Des essais de traction indirects ont fourni des valeurs du coefficient de Poisson dans des conditions de contrainte biaxiale. Les données expérimentales fournies par les essais de compression uniaxiale à vitesse de déformation constante démontrent que le coefficient de Poisson semble dépendre de la déformation, de la vitesse de déformation et de la température. Les courbes de fluage volumétrique obtenues au moyen des essais de compression uniaxiale sous contrainte constante ont permis d'établir une expression liant la déformation volumétrique par fluage et la contrainte appliquée. On a constaté que le coefficient de Poisson était relativement indépendant de l'intensité de la contrainte de compression appliquée pendant le fluage, et pouvait s'exprimer sous forme de fonction puissance de la déformation axiale. Quant aux états de contrainte pluriaxiale, dans lesquels des contraintes de traction et de compression apparaissent simultanément sur les axes principaux, les résultats des essais de traction simple suggèrent que le sable gelé peut présenter un certain degré d'anisotropie, le matériau ayant un comportement différent en présence d'efforts de traction et de compression.

Proc. 4th Can. Permafrost Conf. (1982)

Introduction

Material properties of frozen soils are normally determined from unconfined and triaxial compression tests on cylindrical samples. Advanced analytical techniques, appropriate for design of frozen soil structures, require that data obtained from such tests be extended to provide constitutive relationships for more complex stress states. The von Mises flow law and the assumption of volume constancy have usually been applied to develop such equations. Most studies on the rheology of frozen soils under uniaxial stress fields have not considered the relationship between Poisson's ratio and strain. Data defining the volumetric strain behavior and Poisson's ratio during shear are required to evaluate the applicability and accuracy of existing constitutive expressions.

As part of a laboratory study on the mechanical properties of frozen silica sand, volumetric strain data was obtained for a series of uniaxial constant strain-rate and constant stress (creep) compression tests. In addition, tensile tests on frozen sand, using the split-cylinder (indirect tensile) method, permitted computation of an average Poisson's ratio using a technique developed by Hadley *et al.* (1971). Experimental data show changes in volumetric strain and Poisson's ratio as a function of axial strain for the constant strain rate tests and in terms of time for the creep tests. Temperature effects on Poisson's ratio are shown by additional experimental data. Data analysis shows that Poisson's ratio can be expressed as a function of strain or in terms of stress and time using a power relationship.

Background Information

Poisson's ratio for cylindrical samples in a uniaxial stress field may be expressed as

$$[1] \quad \nu_t = -\frac{d\varepsilon_r}{d\varepsilon_a}$$

where ν_t is the tangent Poisson's ratio, ε_r is the radial strain of the sample, and ε_a is the axial strain. For samples tested in triaxial pressure cells, the radial strains are not normally measured. With zero confining pressure the volumetric strain during shear may be determined by measuring the fluid volume expelled from the cell as the test progresses. For increments of axial stress, values of Poisson's ratio may be calculated as shown by Daniel and Olson (1974):

$$[2] \quad \nu_t = -\frac{\Delta\varepsilon_r}{\Delta\varepsilon_a} = -\frac{\Delta(\varepsilon_v - \varepsilon_a)}{2\Delta\varepsilon_a}$$

where the volumetric strain ε_v (positive for compression) is the ratio of the change in sample volume to the original sample volume over the stress interval and Δ is an incremental change. This expression is valid for a uniaxial stress field and both constant strain-rate and constant stress (creep) tests conducted on cylindrical soil samples.

For indirect tensile tests an average value for Poisson's ratio up to rupture may be obtained using techniques based on the theory of elasticity. A complete solution for the stress distribution in a disk or cylinder subjected to distributed compression loads over finite arcs (Figure 1) was given by Hondros (1959) for a homogenous, isotropic, and linearly elastic material. Hadley *et al.* (1971) used this solution to develop a technique for estimating the tensile strain and Poisson's ratio. The technique requires that both horizontal and vertical deformations be measured during testing. Anagnos and Kennedy (1972) presented simplified expressions for calculation of Poisson's ratio including

$$[3] \quad \nu = \frac{0.0673\,DR - 0.8954}{-0.0294\,DR - 0.0156}$$

for a 4-inch (101.6 mm) diameter sample. *DR* is the slope of the best fit least-squares line for the plot of vertical sample deformation *versus* horizontal sample deformation up to failure.

Experimental Program

Commercially available Wedron sand selected for this study consisted of subangular quartz particles with a specific gravity of 2.65, a uniform gradation (particle size range from 105 to 595 μm), and a uniformity coefficient $D_{60}/D_{10} = 1.50$. The sand volume fraction of 0.64, selected for convenience in sample preparation, gave a dense soil mass and was comparable to densities commonly encountered at field sites. It also was well above the critical volume fraction (0.42) reported by Goughnour (1967) as the point where interparticle friction begins to contribute significantly to the compressive strength of frozen sand.

All samples were prepared in split aluminum molds with amounts of oven-dry sand predetermined so as to give the desired sand volume fraction. To insure a high degree of saturation, molds were partially filled with distilled water and dry sand was slowly poured into the molds as air bubbles escaped to the surface. Mold sides and bottoms were tapped until the desired packing was achieved. All samples were frozen and stored at -15°C for at least 12 hours. Samples were trimmed and weighed prior to mounting for tests and oven-dry weights were determined after testing. Sand volume fractions varied from 0.63 to 0.65 and the degree of ice saturation ranged from 0.96 to 0.99.

Uniaxial Compression Tests

Uniaxial constant strain-rate and constant stress (creep) compression tests were conducted in the triaxial cell illustrated in Figure 2. Axial deformation and loads during the test were monitored using displacement and load transducers. The sample volume change, based on fluid expelled from the cell during sample deformation and measured using a burette (*see* Figure 2), divided by the initial volume gave the volumetric strain at selected increments of deformation. Sample temperatures were maintained by immersion of the triaxial cell in a circulating, low-temperature coolant bath containing a mixture of ethylene, glycol, and water. Temperatures, measured adjacent to the sample with a thermistor, did not vary by more than ±0.05°C. Rubber jackets protected frozen sand samples from contamination by the bath coolant liquid.

Constant strain rate compression tests were conducted on cylindrical samples, 35.7 mm in diameter and with a length/diameter ratio of 2, using a Wykeham-Farrance (model WF-10050) variable-speed test machine. Applied strain rates for eight tests ranged from 8.3×10^{-5}/s to 8.1×10^{-7}/s and temperatures of −2, −6, and −15°C. Creep tests, conducted at −6°C, were performed on samples

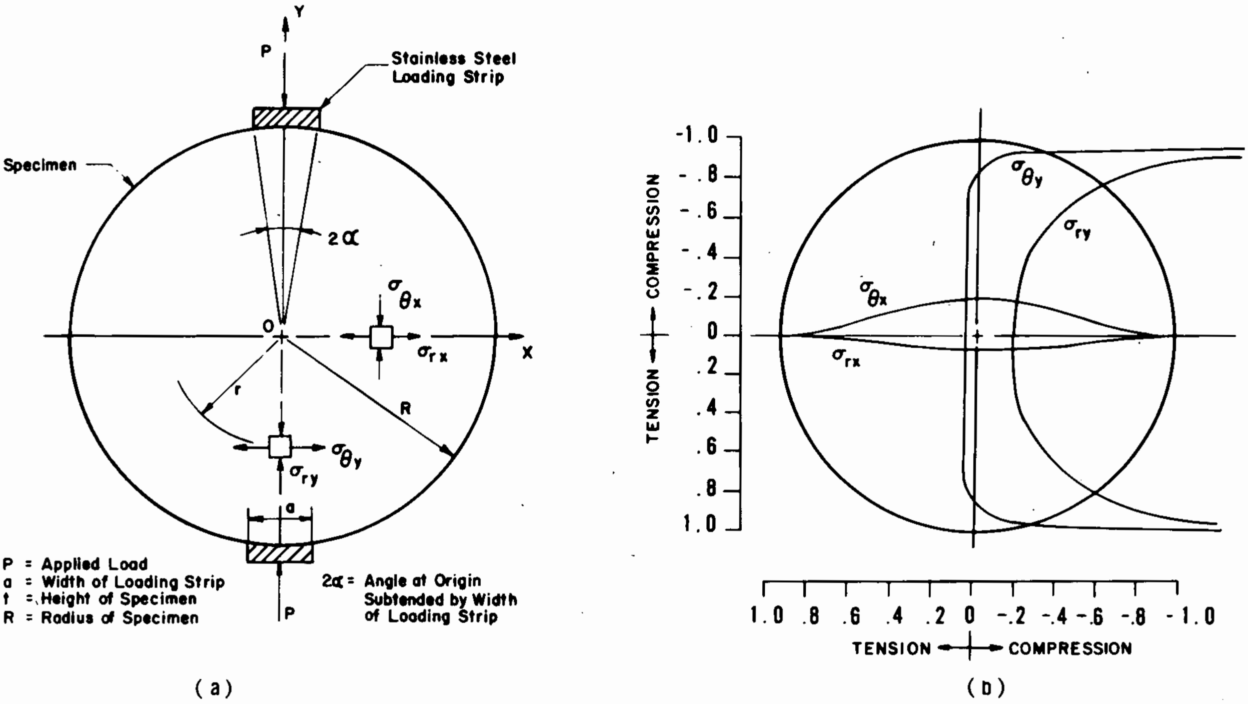

FIGURE 1. Indirect tensile test. **a** Loading conditions. **b** Stress distribution (*after* Hondros 1959).

28.9 mm in diameter (length/diameter ratio of 2) at constant axial stress levels of 1140, 1244, and 1296 psi (7.86, 8.58, and 8.94 MN/m^2). A dead-weight and hanger system was used to apply the stresses, and small increments of weight were added to correct for increased cross-sectional areas with sample deformation.

Indirect Tensile Tests

Indirect tensile tests were conducted using specially designed loading apparatus (Figure 3). The compressive load, measured with a 10,000-pound (44.48 kN) force transducer, was applied to opposite sides of a sample 101.6 mm in diameter (50.8 mm thick) through two steel loading strips with a radius the same as that of the sample (*see* Figure 1**a**). Vertical deformation across the loaded diameter was measured using a displacement transducer and horizontal deformation by two cantilevered arms (fitted with resistance gauges) which could be adjusted to rest on the sample sides (*see* Figure 3). Sand–ice samples were protected from the ethylene-glycol water coolant by a rubber jacket extending between the upper and lower loading plates. Details on fabrication and use of the special loading apparatus are given by Bragg (1980). Temperatures were maintained as described for the uniaxial compression tests.

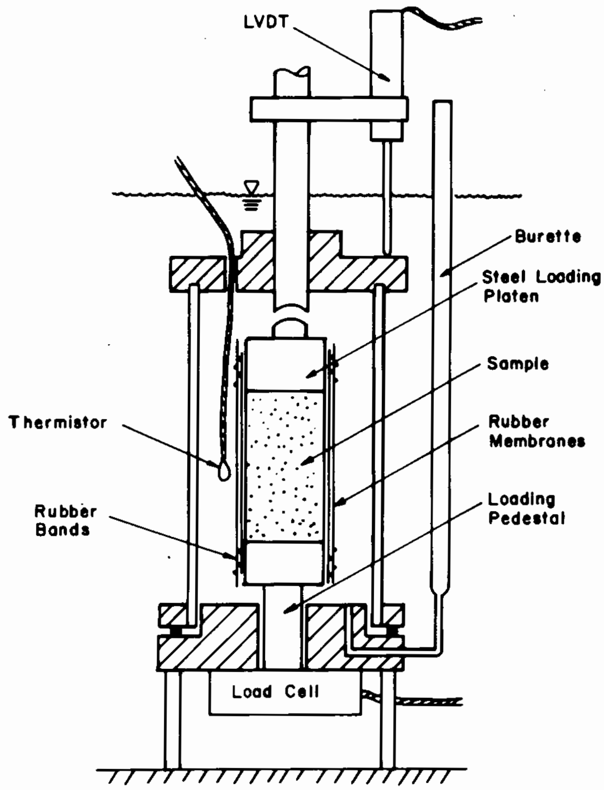

FIGURE 2. Schematic diagram of the triaxial cell.

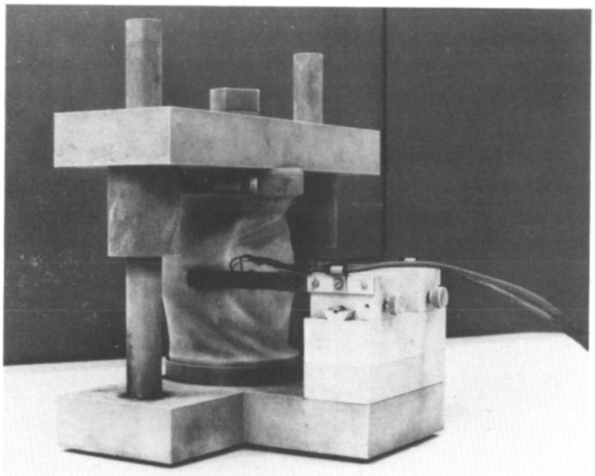

FIGURE 3. Loading apparatus and horizontal deformation transducer for the indirect tensile test.

Interpretation of Results

Results of the experimental program are summarized and discussed according to test conditions: constant strain-rate compression tests, constant stress (creep) tests, and indirect tensile tests.

Constant Strain-Rate Compression Tests

Data for the constant strain-rate tests at a temperature of $-2°C$, summarized in Figure 4, show that volumetric strains and tangent Poisson's ratio are functions of the observed axial strain for a given temperature and strain rate. The influence of temperature on test results for samples at about the same strain rate is shown in Figure 5. The observed volumetric strain behavior is consistent with data reported by Akagawa (1980) and Goughnour and Andersland (1968). As shown *in* Figure 4, the sample volume decreased slightly at low strains and approached a minimum at strains corresponding to the initial yield (limit of initial linear portion of the stress–strain curve). With higher strains the sample volume increased gradually to values greater than the initial sample volume. The corresponding plots of tangent Poisson's ratio indicate a rapid increase in ν_t to about 0.5 at or near the initial yield point. With higher strains a more gradual increase in ν_t was observed. The increase in sample volume and Poisson's ratio appears to continue at axial strains in excess of the failure strain corresponding to the peak stress.

The initial reduction in volume, up to the initial yield point, appears to be associated with compression of the ice matrix, pressure melting of ice at points of contact with particles, and a simultaneous increase in density of the sand particles. The initial

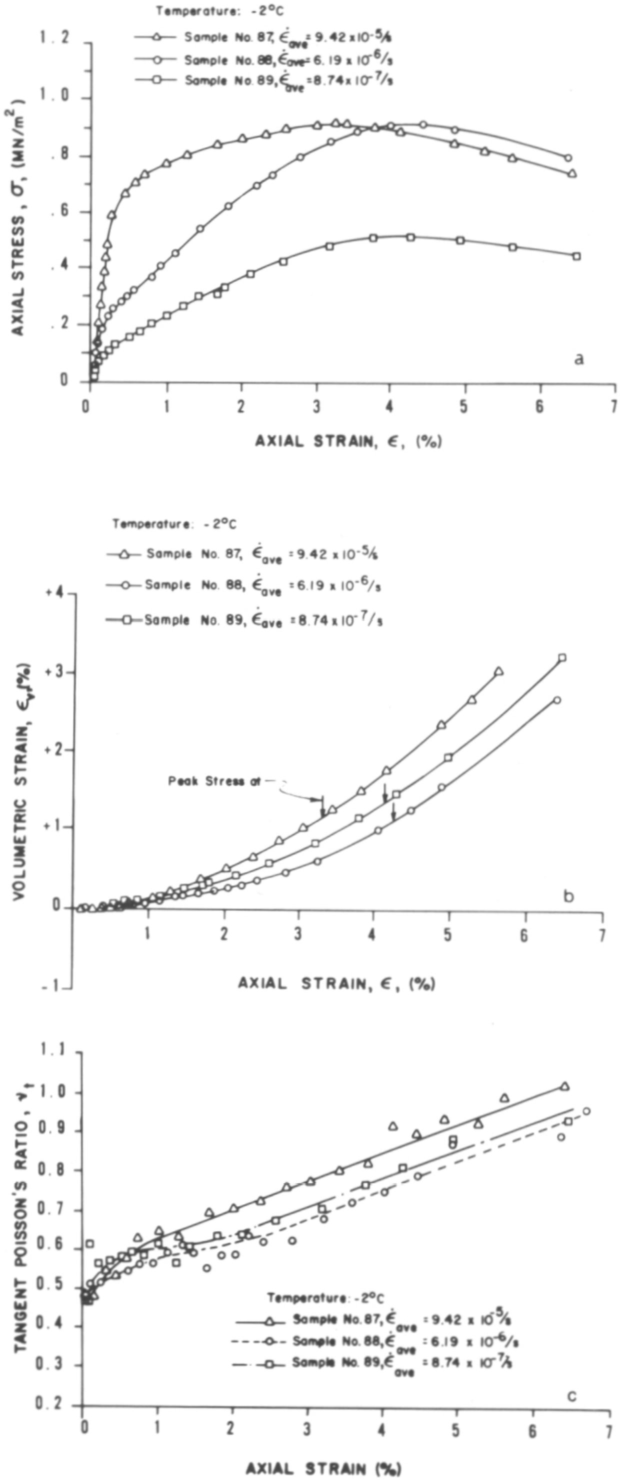

FIGURE 4. Constant strain-rate compression test results at $-2°C$ (*after* Bragg 1980). **a** Stress vs. axial strain. **b** Volumetric strain vs. axial strain. **c** Tangent Poisson's ratio vs. axial strain.

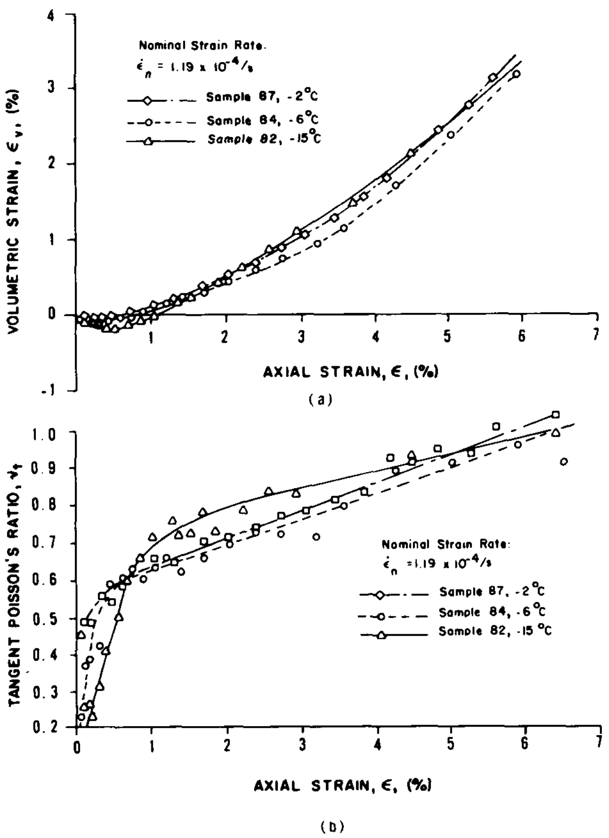

FIGURE 5. Temperature effect on constant strain-rate compression test results (*after* Bragg 1980). **a** Volumetric strain vs. axial strain. **b** Tangent Poisson's ratio vs. axial strain.

yield may occur as the pore ice reaches its yield stress and as interparticle friction and dilatancy begin to contribute to the sample shear strength. This process results in the observed increase in volumetric strain for values greater than the initial yield strain. As would be expected, the computed values of tangent Poisson's ratio varied from less than 0.5 (indicating a volume decrease) at low strain values to greater than 0.5 (volume increase) at strains above the initial yield strain.

The influence of strain rate on the volumetric strain and tangent Poisson's ratio may be observed (*see* Figure 4). Both the volumetric strain and Poisson's ratio appear to vary somewhat with strain rate. As the applied strain rate was increased the samples, in general, experienced a greater volume reduction (lower tangent Poisson's ratio) at low strain values and a greater volume increase (higher tangent Poisson's ratio) after the initial yield. This trend was more evident for samples tested at −6 and −15°C. At higher temperatures (−2°C) the volumetric strain and tangent Poisson's ratio appeared to be less dependent on strain rate. Data summarized (*see* Figure 5) show that for axial strains greater than about 1.5 per cent, the volumetric strain and tangent Poisson's ratio appeared to be relatively independent of temperature.

Constant Stress (Creep) Tests

Data for constant stress compression (creep) tests, conducted at −6°C, show typical creep curves (Figure 6) for the sand–ice material. Each curve includes an instantaneous axial strain immediately after load application followed by regions of primary (decreasing strain rate), secondary (constant strain rate), and tertiary (increasing strain rate) creep. Sample volume change measurements resulted in a series of volumetric strain *versus* time curves similar in shape to the normal creep curves. Sample volumes increased with time after application of the axial stress such that periods of primary, secondary, and tertiary behavior could be defined for the volumetric strain creep curves.

A plot of logarithm of volumetric strain rate against the logarithm of axial stress (Figure 7) suggests that a linear relationship exists. A power law expression, of the form given by Ladanyi (1972) for axial creep, may be used to relate volumetric strain during secondary creep to the applied axial stress as a function of time (t):

$$\varepsilon_V = \varepsilon_V^i + \dot{\varepsilon}_V^c t$$

$$[4] \qquad \varepsilon_V = \left[\frac{\sigma}{\sigma_k}\right]^k + \left[\frac{\sigma}{\sigma_c}\right]^n t$$

where ε_V^i is the pseudo-instantaneous plastic strain achieved immediately after application of the uniaxial stress, σ_k is a volumetric deformation modulus evaluated at an initial strain of 1.0, $\dot{\varepsilon}_V^c$ is the volumetric creep strain rate during secondary creep, n is the volumetric creep parameter, and σ_c is a volumetric creep proof stress evaluated at a volumetric creep strain rate of 1.0/s. Values for the various terms are given (*see* Figure 7).

Tangent Poisson's ratio was close to 0.5 immediately after stress application and appears to be a function of time thereafter. The value of ν_t increased with time, approaching a nearly constant value during secondary creep. Both the rate of increase in ν_t and the magnitude during secondary creep increased, as the applied stress increased. Both tangent Poisson's ratio and volumetric strain are plotted as a function of the axial strain in Figure 8 for the creep tests. The volumetric strain increased with axial strain in a manner similar to results for the constant strain

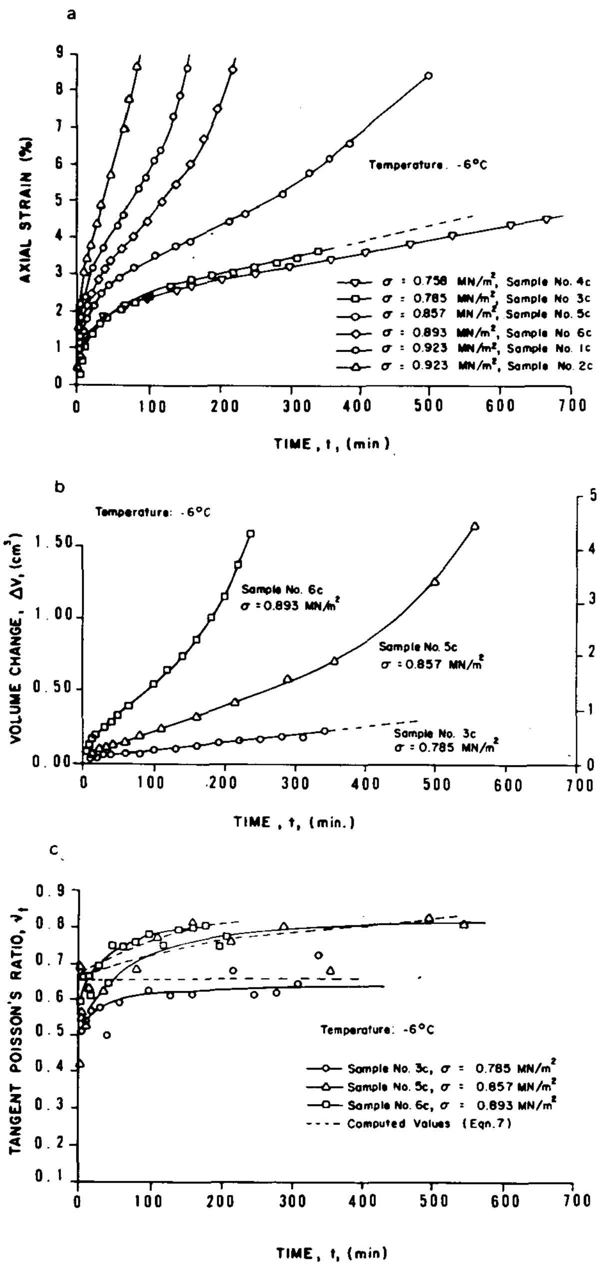

FIGURE 6. Constant stress compression (creep) test results (*after* Bragg 1980). **a** Axial stress vs. time. **b** Volume change vs. time. **c** Tangent Poisson's ratio vs. time.

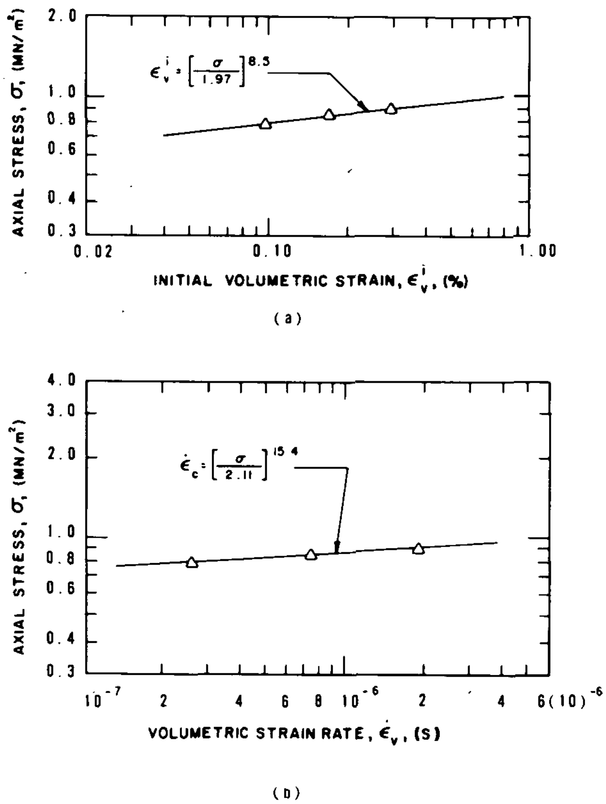

FIGURE 7. **a** Instantaneous volumetric strain vs. axial stress. **b** Volumetric strain rate vs. axial stress.

rate tests (*see* Figure 4). As before, the net volume increase suggests that dilatancy and interparticle friction contribute to creep resistance of the sand-ice material. Tangent Poisson's ratio, plotted as a function of axial strain (*see* Figure 8) appears to be relatively independent of the stress level. The conclusion is that axial strain is required to mobilize dilatancy, interparticle friction, and the corresponding volumetric strain. Therefore Poisson's ratio must be primarily dependent on axial strain level, with values of ν_t greater than 0.5 a result of material dilatancy.

A least-squares regression analysis indicates that the expression

[5] $$\nu_t \simeq B(\varepsilon_a)^m$$

appears to predict ν_t in terms of the axial strain ε_a. The constant B is the ν_t intercept at $\varepsilon_a = 1.0$ and m is the slope for a linear best fit plot of logarithm of ν_t *versus* logarithm of ε_a. Numerical values of B and m for Wedron sand are given (*see* Figure 8). If axial strain during secondary creep can be related to stress and time as proposed by Ladanyi (1972), tangent Poisson's ratio, as a function of time (t), becomes

[6] $$\nu_t = B(\varepsilon_a(t))^m$$

$$\nu_t \simeq B\left[\left(\frac{\sigma}{\sigma_k}\right)^k + \left(\frac{\sigma}{\sigma_c}\right)^n t\right]^m$$

where the creep curves (*see* Figure 6) are approximated by straight lines and the material constants σ_k, k, σ_c, and n for the sand-ice material are given in

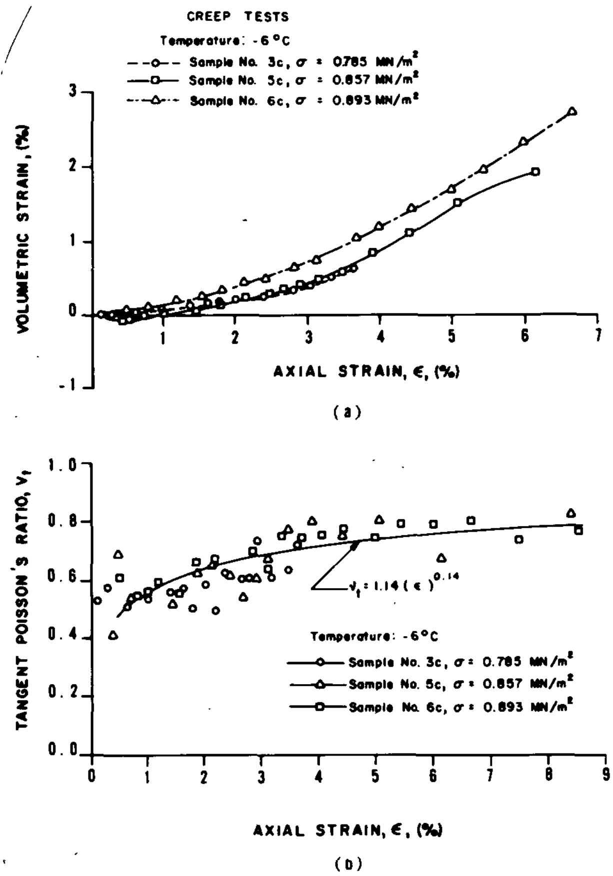

FIGURE 8. Constant stress (creep) test results (*after* Bragg 1980). **a** Volumetric strain vs. axial strain. **b** Tangent Poisson's ratio vs. axial strain.

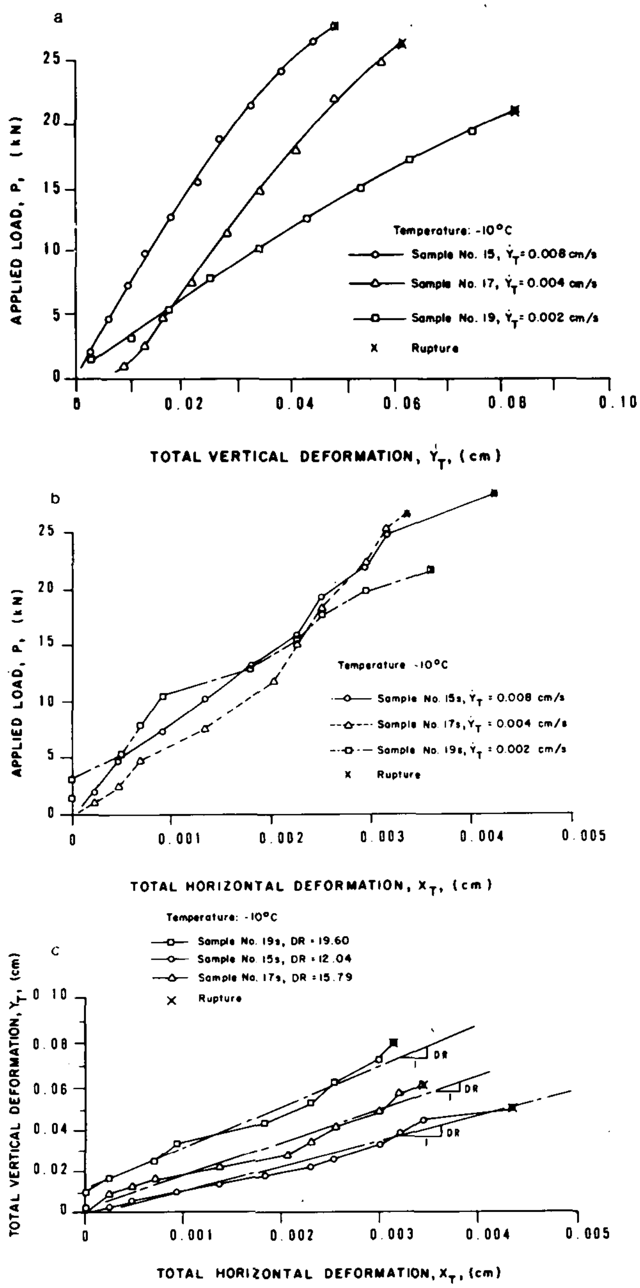

FIGURE 9. Indirect tensile test results (*after* Bragg 1980). **a** Applied load vs. total vertical deformation. **b** Applied load vs. total horizontal deformation. **c** Vertical vs. horizontal deformation.

Table 1. The computed values are in reasonable agreement with the experimental data during secondary creep (*see* Figure 6). Values of ν_t are overestimated during primary creep because linear approximations are used in developing expressions for the axial creep strain (Ladanyi 1972). Other expressions, which account for change during primary creep (Andersland *et al.* 1978; Hult 1966; Odqvist, 1966) can be written.

Indirect Tensile Tests

Typical load-deformation curves, summarized in Figure 9 for the split-cylinder tests, remained nearly linear up to the rupture load. Likewise, the plot of total vertical deformation *versus* total horizontal deformation was approximately linear. The linearity of the curves indicates a brittle, deformation fracture behavior with a nearly linear stress–strain relationship. These results imply at least partial compliance with elastic behavior, hence it appears reasonable to assume that the sand–ice material approximates a homogenous, linearly elastic, isotropic material. The ice crystals should be randomly oriented because of the presence of sand particles and freezing directions which were not explicitly controlled. Careful sample preparation prior to freezing produced a uniform sand density throughout the sample. For these com-

TABLE 1. Material constants for uniaxial compression (creep) tests on Wedron sand

Material constant	Experimental value
σ_c	27.07MN/m^2
n	11.11
σ_k	958.4 MN/m^2
k	8.02

bined factors, the assumption of isotropy and homogeneity seemed reasonable. This permitted use of equation 3 to compute an average value for Poisson's ratio up to rupture from the indirect tensile test data.

Computed values of Poisson's ratio are summarized in Figure 10 as a function of the applied vertical deformation rate. Numerical values ranged from −0.21 to 0.03. Considerable data scatter, due to the testing technique and test equipment limitations, was observed and no discernable relationship was noted between Poisson's ratio and the applied deformation rate or temperature.

The negative values of Poisson's ratio are the result of large *DR* ratios (equation 3). These values may have resulted from several possibilities. Double cleft failures were observed for the majority of the tests conducted. Formation of the wedges prior to failure could have resulted from stress concentrations in the vicinity of the loading strips and large vertical deformations prior to failure of the samples. This explanation could not be verified during testing since visual observation of the samples was prevented due to submergence of the test apparatus in the coolant tank. It is also possible that the sand–ice composite may be described as a "bimodular" or "different modulus" material. Differences in the deformation processes and material (elastic/plastic) stress–strain behavior in tension and compression for the composite sand–ice material coupled with the biaxial stress condition (tension and compression) developed in the indirect tensile test could create a significant variation from the theoretical conditions. Interparticle friction and dilatancy, which contributed to the strength and deformation resistance of frozen sand in compression, may not contribute significantly to the tensile strength or deformation resistance in tension. The tensile strength and deformation parameters may develop primarily from the ice matrix. Consequently, in multi-axial stress states separate constitutive equations may be required to define the stress–strain relationships for tension and compression.

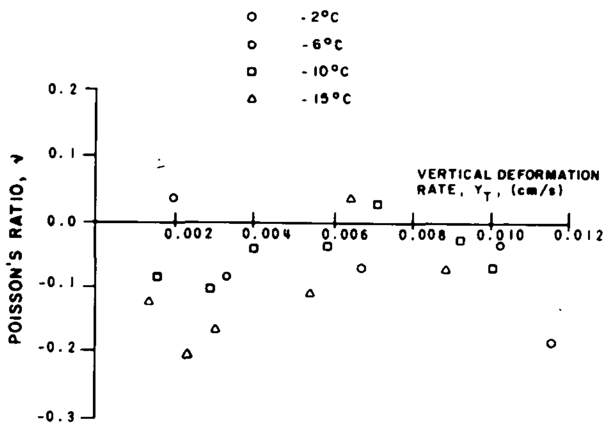

FIGURE 10. Poisson's ratio versus applied vertical deformation rates for the indirect tensile test (*after* Bragg 1980).

Conclusions

1. The volumetric strain and Poisson's ratio determined from uniaxial, constant strain-rate compression tests were observed to vary with axial strain. The magnitude of volumetric strain and Poisson's ratio appears to be influenced by strain rate and temperature. Sample dilatancy appears to be responsible for the volume increase (with Poisson's ratio greater than 0.5) for strains above the initial yield point in the stress–strain curve.

2. Volumetric creep curves, for constant stress compression (creep) tests, permitted development of an expression relating volumetric creep strain to applied stress. Poisson's ratio was observed to be relatively independent of applied stress level and may be expressed as a power function of axial strain. Experimental data indicate that dilatancy contributes to the creep resistance during primary, secondary, and tertiary creep giving values for Poisson's ratio greater than 0.5.

3. Poisson's ratio obtained from indirect tensile data varied from −0.21 to 0.03. No relationship between applied deformation rates or temperature was observed. The negative values of Poisson's ratio may suggest a bimodular material. Differences in the stress–strain properties in tension and compression appear to exist for the composite sand–ice material.

Acknowledgements

This study was supported by National Science Foundation Grant No. CME-7915784 and by the Division of Engineering Research, Michigan State University, East Lansing, Michigan. Their support is gratefully acknowledged.

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