THEORETICAL MODELLING OF MASSIVE ICY BEDS

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Abstract

This paper reviews some of the conditions required to develop massive ice lenses in freezing soils. The wide range of existing soils has been separated into two distinct classes: frost susceptible soils such as silts and clays and coarser material (non-frost prone) such as sands and gravels.

Massive ground ice resulting from processes associated with freezing migratory water during a stationary frost front are described for a geological time scale using an analytical model for the freezing of a semi-infinite medium. The maximum thickness of the massive icy beds in frost susceptible soils is controlled by the segregation freezing temperature and the overburden pressure. The theoretical model showed that the formation and growth of a major ice body in coarser soils required artesian conditions in order to maintain a stationary frost front. This finding supports the view of Mackay (1973) that massive bodies of segregated ice may grow in a manner similar to that of pingos.

Résumé

L'on présente dans cet article les conditions nécessaires à la formation de lentilles de glace massives dans les sols soumis au gel. Le vaste éventail de sols a été séparé en deux groupes: les sols susceptibles au gel tels que les limons et les argiles et les sols granulaires (non susceptibles au gel) tels que les sables et graviers.

On utilise un modèle analytique pour le gel dans un milieu semi-infini avec une échelle de temps géologique pour prédire la formation de lentilles de glace massives lorsque le front de gel est stationnaire. L'épaisseur maximale d'une lentille de glace dans les sols susceptibles au gel est controllée par la température au front de ségrégation et par la pression des terres susjacentes. Le modèle théorique a démontré que la formation de lentilles de glace massives dans les sols granulaires nécessitait une condition de pression artésienne afin que le front de gel soit stationnaire. Ces résultats sont en accord avec les idées de Mackay (1973) voulant que les lentilles de glace massives croissent de la même manière que les pingos.

Introduction

In recent years, primarily as a result of oil exploration and pipeline engineering studies in the Arctic, data on ground ice have accumulated rapidly. Drill-hole records show that thick , tabular, and irregular bodies with much ice, referred to as massive icy beds underlie some areas of the Arctic. Figure 1 shows data on the thickness of 560 massive icy beds in the western Canadian Arctic as compiled by Mackay (1973). The tops of most of the icy masses lie 10-20 m below the ground surface, but some have been encountered at depths of 45 m, which was the lower limit of most drill holes.

According to Mackay (1973), the material that overlies the massive ice consists mainly of stony clay, clay, and sand and gravel. The material that underlies the massive icy bodies is remarkably uniform, being mostly sand and gravel. The origin of these massive icy bodies is not yet fully understood. Ice fabrics, geochemistry and stratigraphic relations suggest that most ice is not buried glacial ice, but of segregated origin. Mackay (1973) proposed that the mechanism of massive ice formation is analogous to that of pingo growth, where the water source is generated by water expulsion resulting from freezing of adjacent coarse-grained sediments over a considerable area.

The purpose of this paper is to present a theoretical model for the formation of massive icy beds under thermal steady state conditions and to discuss the influence of soil type on the likelihood of the formation of massive segregated ice bodies.

Background

When a wet soil is subjected to freezing, the liquid-solid phase change involves two simultaneous processes. Pore water freezes in-situ and water supplied from the unfrozen soil freezes at the segregation-freezing front. Considering continuity of heat flux at the segregation-freezing front, the following equation is derived for one-dimensional heat flow (Harlan, 1974; Loch, 1979; Konrad, 1987):

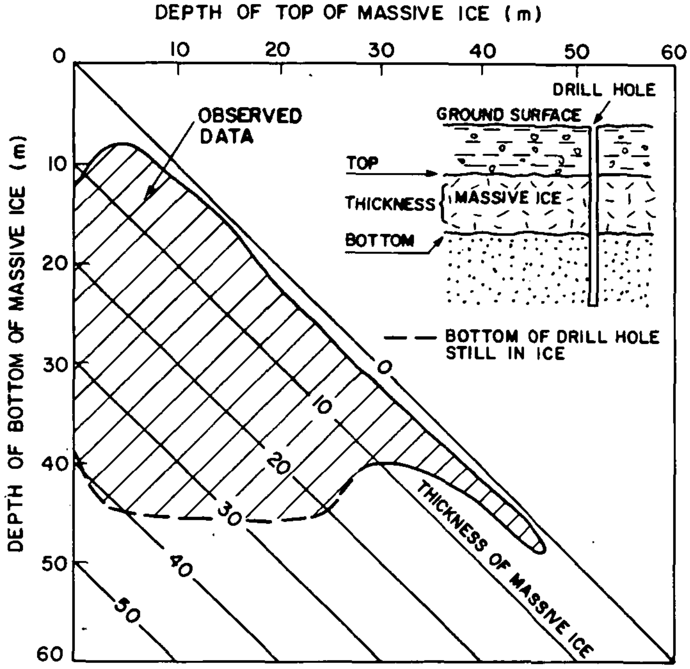

Fig. 1. Massive ice in the western Arctic (After Mackay, 1973)

$$K_f.\mathrm{Grad}\ T_f - K_u.\ \mathrm{Grad}\ T_u = v.L + w.L.dX/dt \quad (1)$$

where K_f, K_u represent the thermal conductivities of the frozen and unfrozen layers

v = water migration rate

w = initial volumetric moisture content of the soil

L = latent heat of fusion of pure water (334 J/cm^3)

dX/dt = frost penetration rate

Grad T = temperature gradient

When the thermal balance is such that no further in situ pore water freezing is required, an ice lens grows at a rate given by:

$$v = (K_f.\mathrm{Grad}\ T_f - K_u.\ \mathrm{Grad}\ T_u)/L \quad (2)$$

Theoretical model for a semi-infinite uniform layer

Transient heat flow is initiated when a sudden temperature drop to below freezing is applied to the ground surface. The frost front progresses into the soil at a rate that is a function of the imbalance between the heat supplied to the front and the heat removed. During transient freezing, frost heave can be computed using a finite-difference scheme and the concept of the segregation potential as outlined in Konrad and Morgenstern (1984). When sufficient time for transient freezing to be completed is available, the frost front becomes stationary and a major ice lens may grow. It is of particular interest to develop a model for these conditions.

At the onset of the formation of the final ice lens, at time $t = t_s$, the idealized problem is depicted in Fig. 2a. The soil-water-ice system is approximated as a semi-infinite medium subjected to an uniform initial temperature, T_o. To initiate the freezing process, the outer surface temperature is dropped to T_c, below freezing, at time $t = 0$ and held constant throughout the freezing process. The position of the frozen-unfrozen interface at time ts is given by X_s and the water flux by v_s. To analytically model this system, several simplifying assumptions were made:

1) The medium is composed of homogeneous layers
2) Constant thermophysical properties are assumed for each layer
3) In the frozen zone, the temperature gradient is linear
4) In the unfrozen layer, the temperature field must satisfy Fourier's equation:

$$\partial T/\partial t = \alpha\, \partial^2 T/\partial x^2 \quad (3)$$

The transient response of a one-dimensional semi-infinite medium, initially at T_o throughout, and whose temperature at the frost front is maintained at 0 °C, is given by (Carslaw and Jaeger, 1959):

$$T(x,t) = T_o \mathrm{erf}\left(x / \sqrt{\alpha t}\right) \quad (4)$$

where erf(x) is the error function defined as

$$2/\sqrt{\pi}\int_0^x e^{-u^2} du$$

The temperature gradient at the frost front is thus obtained as the limit of the derivative of eqn. (4) with respect to x when x approaches zero which is given by (5):

$$dT/dx = GradT_u = T_o / \sqrt{\alpha \pi t} \quad (5)$$

At the onset of a stationary frost front, the heat balance is thus expressed as:

$$K_f \cdot T_c / X_s = \left(K_u \cdot T_o / \sqrt{\alpha\ \pi\ t_s}\right) + v_s \cdot L \quad (6)$$

The rate of growth of the segregated ice lens at a stationary frost front is calculated from equation (7) which is equation (2) for a layered system composed of the frozen layer, the segregated ice and the unfrozen soil as depicted schematically in Fig. 2b.

$$v(t) = \left[K_f K_i T_c / \left(K_f e(t) + K_i X_s\right) - \left(K_u \cdot T_o / \sqrt{\alpha \pi t}\right)\right] / L \quad (7)$$

where $$e(t) = 1.09\int_{t_s}^{t} v(t)dt$$

K_i = thermal conductivity of ice

For saturated frost susceptible soils with an unlimited water supply and under hydrostatic water conditions, the ultimate thickness of this ice lens is obtained when the pore water pressure at the base of the segregated ice mass reaches the hydrostatic pressure. Using the Clapeyron equation for solute free ground water, one readily infers the temperature, T_e, for which the pore pressure is equal to the hydrostatic

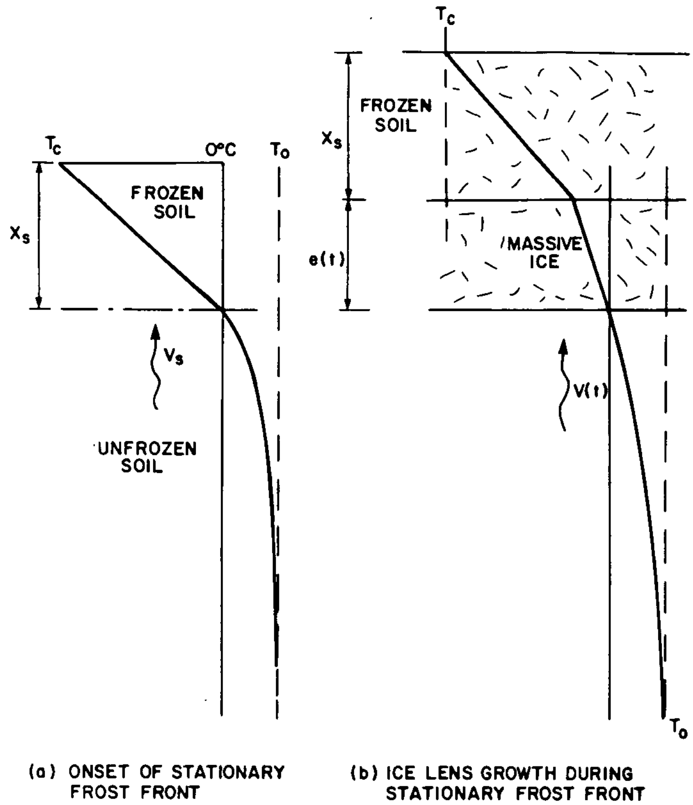

Fig. 2. Schematic of the temperature distribution in the semi-infinite domain

pressure as a function of overburden pressure, P_o (Konrad and Morgenstern, 1982):

$$T_e = -P_o V_i T^*_o / L \tag{8}$$

where T^*_o = temperature of the freezing point of pure water (273.15 °K)

V_i = specific volume of ice

With consistent units Equation (8) reduces to:

$$T_e = -0.89 P_o \tag{9}$$

where P_o is expressed in MN/m² and T_e in °C.

It is noted that Eqn. (8) is different from the relationship between freezing point depression and pressure given by Edelfson and Anderson (1943), assuming that the total change in the pressure on the ice is always equal to the total change in pressure on the water. These conditions then yield a slope dT/dP of approximately - 0.075 °C/MPa.

The overburden pressure is calculated from:

$$P_o = \gamma f X_s + \gamma i\, e(t) \tag{10}$$

where γf and γi are the unit weights of frozen soil and ice, respectively.

The segregation-freezing temperature, T_s, at the onset of stationary frost front is dependent upon soil type and applied pressure (Konrad and Morgenstern, 1982b; Konrad, 1988). In general, T_s decreases with increasing fine content as well as with increasing overburden pressure. A particularly important assumption is that the temperature at the base of the ice lens remains constant during its growth. This, in turn, means that the end of growth is reached when T_e (Eqn. (8)) becomes equal to T_s. This will be explored further in the next section with the aid of numerical examples.

Formation of massive ice in frost susceptible soils

It is well known that the relationship between unfrozen water content (and hence frost susceptibility) and temperature for soils of different textures depends mainly on specific surface of the soil, amount of fines, type of minerals, pressure in the ice and water phases, solute concentration, and the void ratio. In frost susceptible soils, the contribution of segregation freezing is significant, if not outweighing in situ freezing. During the advancing frost front phase or transient freezing, Konrad and Morgenstern (1982a) demonstrated that a given soil can be characterized either by its segregation-freezing temperature and the overall hydraulic conductivity of the frozen fringe, or by its segregation potential, SP. The segregation potential, itself explicable in terms of detailed characteristics of the frozen fringe, is readily obtained from laboratory freezing tests, as it is the ratio of water intake rate and temperature gradient in the frozen soil near the frost front. Frost susceptible soils display values of SP between 200 and 350 10^{-5} mm²/s.°C (Konrad and Morgenstern, 1983).

Let us consider a saturated clay at an initial temperature T_o = +2 °C subjected to a step change of the surface temperature to -20 °C. This simplified temperature boundary conditions does not necessarily model accurately actual climatic changes. However, it can be used as a reference condition and is valuable for the discussion of the results

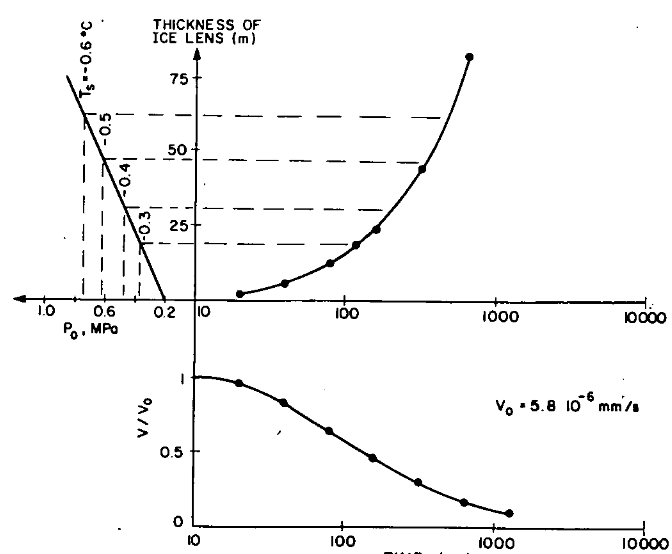

Fig. 3. Prediction of ice lens growth in frost susceptible soils The following values were used in the simulation: $\alpha=10^{-4}$ cm²/s; T_c=-20 °C; T_o=2 °C; K_u=1.46 mW/mm°C; K_f=1.76 mW/mm°C; Ki=2.2mW/mm°C; t_o=10 years

obtained with the model in frost-susceptible and coarse-grained soils. Assuming that it takes 10 years to reach a condition of stationary frost front with a water migration rate of 0.5 mm/day (5.8 10^{-6} mm/s), the depth of frozen soil is 12.2m according to Eqn. (6). Figure 3 shows the results of the prediction of ice lens growth with time using Eqn. (7). The soil parameters are indicated on Fig. 3. The model confirms that it is possible to a maintain heat balance at a stationary frost front for a period in excess of 1000 years. Moreover, the water migration rate, hence the rate of ice lens growth, decays with time, which is compatible with laboratory experiments and field observations at the Calgary test facility (Carlson et al,1982). The decrease in water migration rate may be attributed to the decrease in suction potential at the ice lens as the overburden pressure increases with increasing thickness of the ice lens. It is also related to the decrease of net heat extraction rate as a result of temperature changes with time in both frozen and unfrozen layers.

As mentioned above, the end of ice lens growth is related to the overburden pressure and to the temperature of ice lens formation at the end of transient freezing. Since the latter is soil type and pressure dependent, the relationship between overburden pressure and ice lens thickness has been plotted on Fig. 3 as well. The overburden pressure required to stop the growth process has been indicated for values of the segregation-freezing temperature ranging from -0.3 to -0.6 °C. The maximum thickness of the massive ice layer is found to be approximately 18, 31, 47 and 62 m for values of T_s equal to - 0.3, -0.4, -0.5 and -0.6 °C, respectively. The growth period is 120, 210, 335 and 460 years, respectively.

If the surface temperature is still below freezing at the termination of ice lens growth, the frost front penetrates further at a rate dictated by Eqn (1) with a negligible water migration rate.

Formation of massive ice in coarse-grained soils

Saturated coarse-grained soils with little fine content are generally non frost susceptible, i.e. the water migration rate to the frost front is small. In order to simulate the growth of massive icy beds in sands, the model was used with a water migration rate at the onset of a stationary frost front equal to 0.05 mm/day (5.8 10^{-7} mm/s), which is one order of magnitude smaller than for the frost susceptible case. Eqn (6) yielded a depth of frost penetration of 31.2 m after a period of transient freezing of 10 years. However, the assumption that the rate of water migration decreases with time as for the frost susceptible soil did not result in convergence of Eqn. (7) and the frost front had to penetrate further in order to achieve heat flux balance at the interface.

The model was consequently used to seek the value of water migration rate needed to maintain a stationary frost front with time. The results of the analysis (Fig. 4) reveal that the water migration rate to the freezing front increased steadily during the first 100 years, where it reached a maximum value equal to 3.8 times the rate at the onset of stationary frost front. After 300 years of sustained freezing, water migration rate to the ice lens decreased in order to fullfil the conditions of stable position of the growing ice body. After 1000 years of freezing, the water migration rate is, however, still about twice the rate at the onset of stationary frost front. These results suggest therefore that, in order to grow thick icy beds in coarse-grained soils as reported by Mackay (1973), there must be unlimited water supply. Moreover, the pore water must also be under pressure so that the rate of freezing can be dictated by thermal conditions to maintain the frost front stationary.

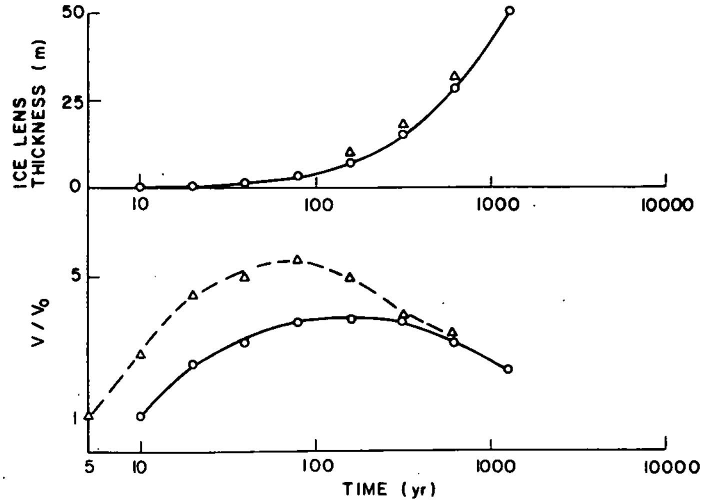

Fig. 4 Prediction of ice lens growth in coarse-grained soils
The following values were used in the simulation:
$\alpha=10^{-4}$ cm^2/s; $T_c=-20$ °C; $T_o=2$ °C; $K_u=1.46$ mW/mm°C; $K_f=1.76$ mW/mm°C; $K_i=2.2$mW/mm°C; $V_o=5.8$ 10^{-7} mm/s
O : $t_o=10$ years, $X_s=31.2$ m
Δ : $t_o=5$ years, $X_s=23.2$ m

Figure 4 also shows the results of the analysis for the case where transient freezing stops after 5 years when the thickness of the frozen soil was 23.2 m. The theoretical migration rate displayed a maximum value equal to 5.6 times the rate at the end of transient freezing. The fact that the water migration rate must increase with time in order to maintain a stationary frost front, and consequently the growth of a single ice mass, suggests that pore water must be under an artesian condition. This was already inferred by Mackay (1973) in order to explain the presence of massive ice beds in coarse-grained material. He suggested that the mechanism of water expulsion in a closed system, which is widely accepted for pingo growth, might be extended to that of massive segregated ice.

The conditions controlling the growth period of a single ice lens in coarse-grained soils is therefore the existence of unlimited supply of water under artesian pressure rather than to the segregation-freezing temperature, which is fairly close to 0 °C in coarse-grained soils. As suggested by Mackay (1973), freezing over extensive areas should provide enough water for the growth of these massive ice bodies. According to Fig. 4, 25 to 50 m thick ice masses require freezing over periods ranging from 550 to 1300 years.

Conclusions

An analytical study is presented for the freezing of a semi-infinite domain of saturated soil with special attention to the growth of the major segregated ice lens when the frost front is stationary for hundreds of years. The results indicate that the growth of massive ice induced by water migration to the freezing front through a frozen fringe is possible in very frost susceptible soils. The rate of water migration at a stationary frost front decreases steadily as a result of decreasing net heat extraction rate at the ice lens and increasing overburden pressure with time. The end of growth is related to the segregation-freezing temperature at the onset of the formation of the ice mass and to the overburden pressure.

The analytical model also indicated that freezing in saturated coarse-grained soils required to have a variable water migration rate in order to maintain the frost front at a stable position. Furthermore, the water migration rate had to increase to values equal to 4-6 times the rate at the onset of stationary frost front over a period of about 100 years and thereafter to decrease steadily with time. This finding is, however, of fundamental interest since it suggests that water must be under artesian conditions in order to satisfy the demand for water needed for heat balance at the base of the growing ice mass. Under these conditions of artesian pressure, the factor controlling the growth of the ice lens is most likely the availability of water, hence of the extent of the area that is being frozen.

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