MODELING THE MOVEMENT OF WATER, HEAT AND SOLUTES IN FROST-SUSCEPTIBLE SOILS

Francisco PADILLA and Jean-Pierre VILLENEUVE

Institut National de la Recherche Scientifique - Eau
2800, rue Einstein, Québec, P.Q., G1X 4N8, Canada.

Abstract

The behavior of frost-susceptible soils is analyzed using finite-element solutions for water flow, heat and solute transport in one-dimension. These solutions take into account the discontinuity concept of the Stefan problem for different conditions of ice in soils. The degrees of freedom of this problem are hydraulic head, soil temperature, and solute concentration. The proposed MELEF-3v model is based on Miller's theory and simulates the presence and formation of ice lenses in saturated and partially saturated soils. The main processes, namely dispersion, convection, adsorption, degradation, overburden and energy exchanges at the soil surface were taken into account. Frost heave of actual pavements is specially considered in applications. Results are very promising when compared to analytical solutions and experimental data.

Résumé

Le comportement des sols gélifs est envisagé à partir de la résolution numérique à une dimension par la méthode des éléments finis de l'écoulement de l'eau et du transport de la chaleur et des solutés. Cette résolution considère le concept de discontinuité du problème de Stefan pour les différentes conditions de la glace dans le sol. Les degrés de liberté du problème sont la température, la pression et la concentration des solutés dans l'eau. Le modèle MELEF-3v est basé sur la théorie de Miller et il est construit pour simuler l'évolution du gel et la formation de lentilles de glace dans les sols saturés et non saturés. Les principaux processus pris en considération sont la dispersion, la convection, l'adsorption, la dégradation, les effets de surcharge et les échanges d'énergie à la surface du sol. Les applications tiennent compte des soulèvements par le gel de chaussées récentes. Les résultats des simulations s'avèrent très encourageants lorsqu'ils sont comparés aux solutions analytiques et aux données expérimentales.

Introduction

The transport of water, heat and solutes in the saturated and unsaturated zones of soil have received considerable attention in contemporary modeling efforts. The simultaneity and inter-relationships of these phenomena are presently of major concern in evaluating the movement of water in frozen soils.

When a frost-susceptible soil freezes, water is strongly attracted towards the frozen zone, usually resulting in accumulations of ice too large to be contained within the soil pores. The excess ice must be accommodated by expansion of the soil matrix. This process, called frost heave, may cause damage to several kinds of soil structures, either directly by differential movement, or indirectly due to unstable soil conditions following thaw.

Ground freezing is responsible for frost heave in permanently or temporarily frozen subsoils. In cold climates, soil freezing and thawing have caused much damage to buildings, pipelines, road and airfield pavements. Water salinity becomes more and more significant for melting salt related phenomena, deep-frozen soils and permafrost problems.

Since frost heave is such an important design consideration for all kinds of pavements, the purpose of this study is to apply the one-dimensional finite-element model MELEF-3v for frost heave of a northern road. This road is maintained with the help of melting salts. The infiltration of water and dissolved salts in the pavement system and in the supporting soil is solved for the numerical solution of water flow, heat and solute transport. Unless uncoupled, these equations are sequentially solved, in the same time step, for phase changes, ice lensing and other parameters, through several solutions to the discontinuity concept of the Stefan problem.

Governing equations

Flow of water

The equation for the transient flow of water in a slightly compressible and partially saturated soil, can be written (Laliberté *et al.*, 1967; Pickens *et al.*, 1979; Padilla and Gélinas, 1988; Padilla and Villeneuve, 1989):

$$\rho_w m(H,T)\frac{(P_w - P_i)}{\tau(T)}\frac{\partial\tau(T)}{\partial t} - \rho_w m(H,T)\frac{\partial P_i}{\partial t} + \rho_w m(H,T)\frac{\partial H}{\partial t} = \frac{\partial}{\partial z}\left\{\rho_w \frac{k(H,T)}{\eta_w(T)}\frac{\partial H}{\partial z}\right\} - i$$

where ρ_w is the water density, T is the temperature, H is the water potential , P_w and P_i are the pressures of water and ice, k is the intrinsic permeability, η_w is the dynamic viscosity of water, m is the specific moisture capacity, τ is a coefficient of capillary conversion, and i is the mass of water changing to ice per unit volume. The above equation is non-linear because of the dependence of m and k on the state variable H.

Heat transport

The equation for the unsteady transport of heat in a partially saturated porous medium, can also be written (Menot, 1979):

$$c(H)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left\{K(H)\frac{\partial T}{\partial z}\right\} - \rho_w c_w \vec{w}\frac{\partial T}{\partial z} + \left((c_w - c_i)T + L_f\right) i \qquad (2)$$

where c is the calorific capacity by unit volume of the porous medium, c_w and c_i are the specific calorific capacity of water and ice, $\vec{w}$ is the Darcy velocity, and K is the thermal conductivity.

Solute transport

Finally, the convection-dispersion of solutes in partially saturated soils can be expressed (Pickens *et al.*, 1979, Padilla and Villeneuve, 1989):

$$(\theta_w + \rho K_a)\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left\{\theta_w D\frac{\partial C}{\partial z}\right\} - \vec{w}\frac{\partial C}{\partial z} - (\theta_w + \rho K_a)K_d C + (1 - X_i)\frac{i}{\rho_w}C \qquad (3)$$

where C is the dissolved solute concentration, θ_w is the water content, D is the hydrodynamics dispersion, ρ is the bulk density of the porous medium, K_a is the adsorption partition coefficient, K_d is the first-order degradation rate, and X_i is the coefficient of solute redistribution during freezing.

The Galerkin technique is used to determine approximate solutions to equations 1, 2 and 3 under the appropriate initial and boundary conditions. In a one-dimensional finite-element domain, hydraulic heads, temperatures and solute concentrations were represented by spatial quadratic functions (fig. 1). Full details of the solution procedures can be found in Padilla *et al.* (1990).

Water, heat and solute balances at the soil surface

Frost heave of pavements is of major concern in taking into account water, heat (Smith, 1979; Akan, 1984) and solute exchanges at the surface of the domain. The upper boundary conditions are obtained from (1), (2) and (3) as:

$$-K\frac{\rho_w}{\eta_w}\frac{\partial H}{\partial z} = q_w \qquad (4)$$

$$-K\frac{\partial T}{\partial z} = (T^* - T)\left[c_w q_w + \rho_a c_a \frac{K_v^2 \vec{v}}{\left[\ln\frac{Z_v}{r}\right]^2}\right] + R_s(1 - \alpha_s) + \varepsilon R_L - \varepsilon\sigma T^4 \qquad (5)$$

$$-\theta D\frac{\partial C}{\partial z} = (C^* - C)\frac{q_w}{\rho_w} \qquad (6)$$

in which the three equations are evaluated at the soil surface. These equations express the mass rate of rainfall q_w, as well as the diffusive fluxes of heat and solutes per unit surface area, respectively. In (4), (5) and (6), ρ_a is the density of air, K_v is the von Karman constant, $\vec{v}$ is the average wind velocity measured at Z_v above of soil surface, r is a length measure of roughness of the soil surface, R_s is the incident solar radiation, as is the albedo, ε is the emissivity, R_L is the incoming long-wave radiation, σ is the Stefan-Boltzmann constant, c_a is the specific heat of air, C^* is the solute concentration in the influx water, and T^* is the external temperature.

When ground surfaces or road and airfield pavements are exposed to cold climates in permafrost zones, solar radiation and/or wind velocity could be significant factors affecting ground freezing phenomena.

Ice formation

The phase changes in porous media are assumed to be produced at a "freezing front" (Menot, 1979). We call "front" a surface where one or several parameters of the model have a discontinuity or a discontinuous derivative. This is pointed

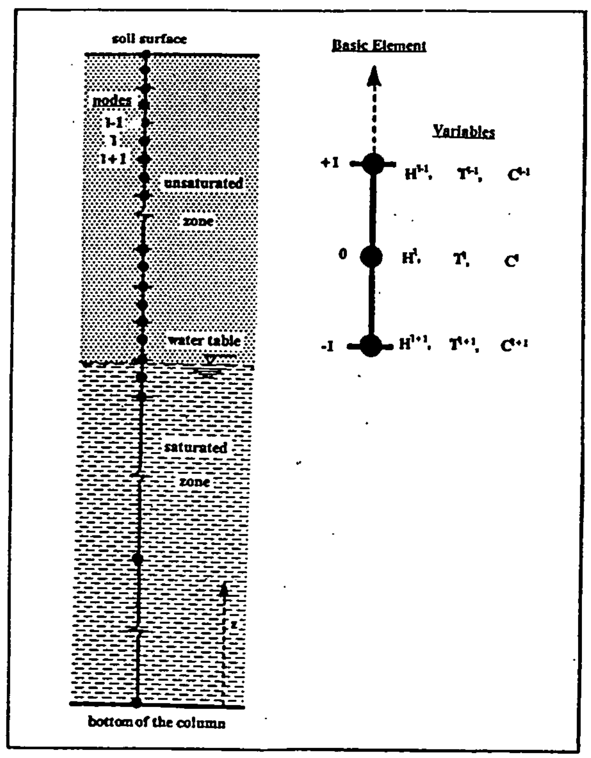

Figure 1 Finite-element discretization of the unsaturated-saturated zone of soil.

out by the Stefan problem. Then, equations (1) to (3) are true in the sense of distributions and they may also be written in terms of a "jump". From this point of view, the volumetric rate of freezing can be expressed as:

$$i = -\frac{\left[K\frac{\partial T}{\partial z}\right]_N}{\left((c_w - c_i)T + L_f\right)\Delta z} \tag{7}$$

where Δz is the length of spatial discretization and $[\ ]_N$ represents the "jump" or variation between the two sides of the discontinuity. Typical discontinuities are: the propagation of a freezing line in a saturated soil (Fremond, 1979) and the growth of ice lenses within forced discontinuities in the frozen soil fabric (O'Neill and Miller, 1985). These lenses of ice are responsible for the upward displacement of the soil surface (frost heave).

In order to consider the thermodynamic equilibrium governing phase changes, four node types can be defined:

Unfrozen. There is no ice and equations (1) to (3) are ordinarily solved.

Frozen, unsaturated. Water begins to freeze in the unsaturated zone if temperature cools down enough to satisfy the following generalized Clapeyron equation (Kay and Groenevelt, 1974; Miller, 1980),

$$\frac{P_w}{\rho_w} - \frac{P_i}{\rho_i} = \left[\frac{L_f}{273.15}\right]T^\circ + \frac{\pi}{\rho_w} \tag{8}$$

where π is the osmotic pressure and T° is the temperature (°C). For unsaturated media, ice pressure can be set equal to zero (atmospheric):

$$\frac{(P_w - \pi)}{\rho_w} = \left[\frac{L_f}{273.15}\right]T^\circ \tag{9}$$

Then, most of the ice would fill up the remaining voids of the porous medium (fig. 2a).

Frozen, saturated. After a frozen node becomes saturated, i.e. without air voids, the ice and water pressures can increase. The higher water pressure reduces or even reverses water flow (Hopke, 1980). The freezing front is then mobile but ice does not move inside the pores (fig. 2b).

In an unfrozen saturated porous medium, water would begin to freeze when temperature cools down enough to satisfy the Clapeyron equation (8) for $P_i = P_w$:

$$P_w - \left[\frac{\rho_i}{\rho_i - \rho_w}\right]\pi = \left[\frac{\rho_i\rho_w}{\rho_i - \rho_w}\right]\left[\frac{L_f}{273.15}\right]T^\circ \tag{10}$$

Frozen, heaving: When the ice and the water pressures increase enough to cancel the effective vertical stress, heaving is allowed (fig. 2c). Then, inside the pores located just below the ice lens, the total vertical stress would be:

$$\sigma_z \leq \chi P_w + (1-\chi)P_i \tag{11}$$

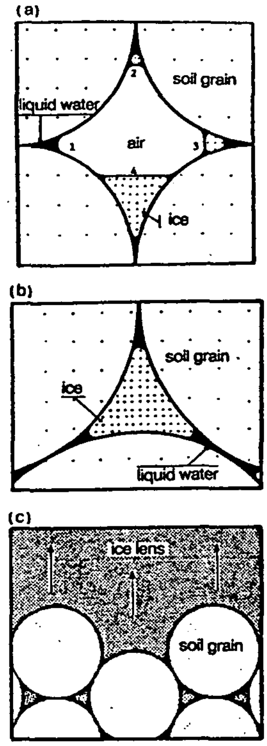

Figure 2. Three hypothetical configurations of ice during ground freezing:
a) Frozen unsaturated (1,2,3,4 = ice evolution stages);
b) Frozen, saturated;
c0 Frozen, heaving.

where $\chi(\rho_w)$ is a weighting factor (O'Neill and Miller, 1985). The ice velocity or rate of heaving at this point can then be calculated by the expression,

$$\vec{i} = \vec{w}\frac{\rho_e}{\rho_g} \tag{12}$$

This latter approach implies an ice discontinuity just below the ice lens, that is to say, ice moves upward in the lens but it does not move inside the pores that are located below the ice lens. Therefore, part of the ice forming in the vicinity of this type of node would increase the ice content of the soil pores at a rate given by the expression:

$$\bar{i} = \frac{i}{\rho_g} - \frac{\vec{i}}{\Delta z} \tag{13}$$

where i is the fraction of ice forming inside the pores of the soil. The present numerical approach is somewhat different from the one proposed by O'Neill and Miller (1985).

Evaluation of the numerical model

Analytical verification

The heat and solute transport portions of the model were tested by comparison with the analytical solution proposed by Ogata and Banks (1961). A formal finite-element approach is used to take into account outflow (Fig. 3) and inflow (Fig. 4) conditions on a boundary (Padilla *et al.*, 1990).

The one-dimensional transient non-linear water flow equation (1) has a suitable analytical solution if a logarithmic dependence is established between saturation and water pressure (Padilla *et al.*, 1988). In this case, Richard's equation of unsaturated flow has a transport form and therefore it can also be compared to the analytical solution of Ogata and Banks (Fig. 5).

Testing the accuracy of numerical schemes used to solve the one-dimensional transient soil freezing processes is limited by the scarcity of suitable analytical solutions.

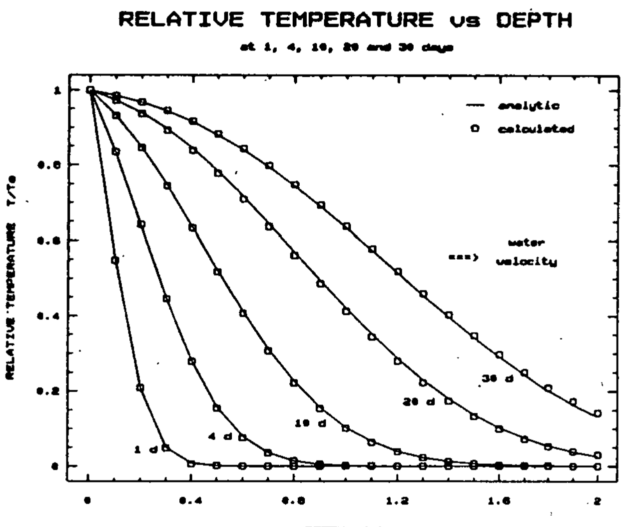

Figure 3. Comparison of numerical model and analytical solution results for convection-dispersion of the heat, when the water velocity is oriented downward. Open boundary conditions are imposed at the bottom of the column.

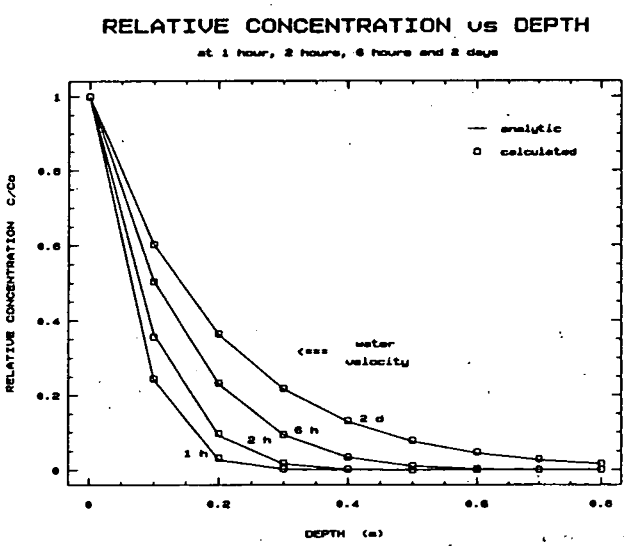

Figure 4. Comparison of numerical model and analytical solution results for the convection-dispersion of a solute, when the water velocity is oriented upward. Cauchy boundary conditions are imposed at the bottom of the column.

SOME THEORETICAL FREEZING FEATURES

When compared to soil freezing phenomena (O'Neill, 1983), the model predict expulsion of water with initial freezing. As time proceeds, liquid flow reverses and water enters the frozen fringe as predicted by the Clapeyron equation. From this thermodynamic point of view, sharp variation of water and ice pressures, phase changes at a freezing front, ice lensing or whatever occurs microscopically, are numerically averaged out at the macroscopic level of the discretized finite-element solution. Other phenomena are predicted during freezing by the model, like solute redistribution or several overburden consequences, as water expulsion when effective stress is still affecting the soil grains. When effective stress cancels, water flow reverses and an ice lens is allowed to break through the soil matrix. Then, while ice pressure in the lens equals overburden pressure, near below in the soil pores, water pressure is governed by the water flow equation and ice pressure follows Terzaghi's theory of soil mechanics (equation (11); Terzaghi and Peck, 1948).

FROST HEAVE OF ROAD PAVEMENTS

Within pavement systems, frost action is manifested by heaving of the surface during freezing and/or a decrease in the load carrying capacity of the pavement during thawing. While actual ice lenses commonly forms in frost-susceptible subsoils, surface pavements subsides during thaw weakened periods. The lowering rate at the surface depends on the rate of melting, the weight of traffic and the road structure resistance. This lowering rate is then equal to or smaller than the melting rate of the actual ice lenses.

Present applications take into account the behavior of a recent road (present Henri IV boulevard, Québec city) surfaced with concrete (Laroche, 1988). Heave and

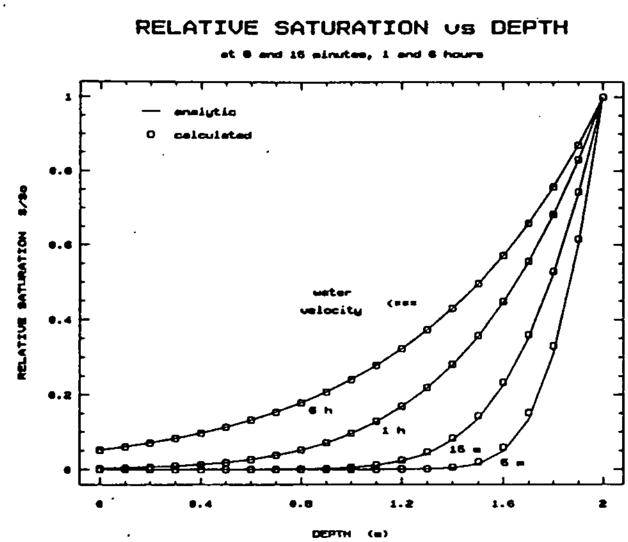

Figure 5. Comparison of numerical model and analytical solution results for water intake from 2 m deep water table towards the unsaturated zone of a initially dry soil.

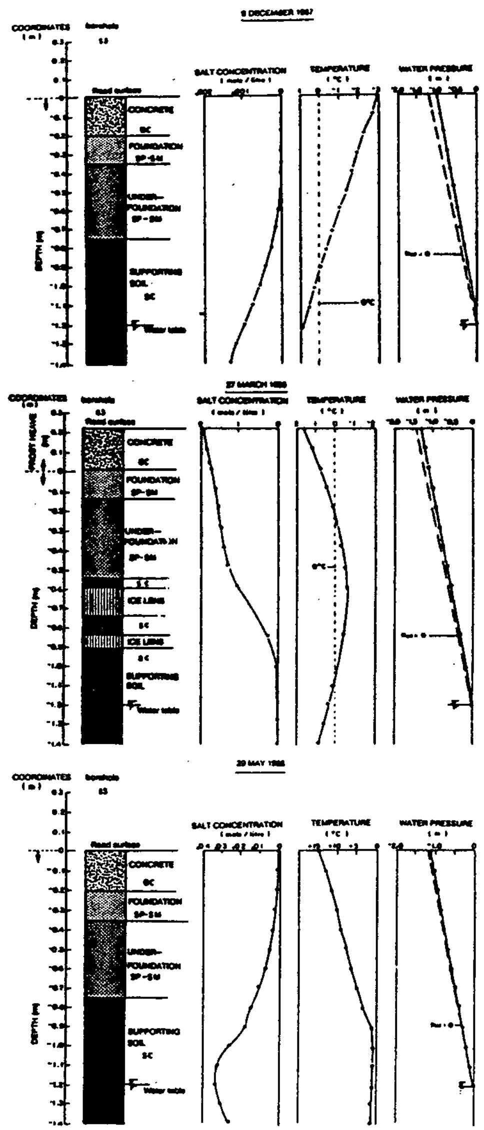

Figure 6. Some freezing features regarding the behavior of a road during a free-thaw period (1987-88 ; S3 borehole ; Henri IV Blvd ; Laroche, 1988).

settlement phenomena because of freezing and thawing periods are specially considered in simulations. Nevertheless, it is well known that the migration of water, heat and melting salts through a particular porous media, during winter and spring times, will to some extend control ice formation and general behavior of frequently frost affected roads. By way of example, ice lensing, water pressure, temperature and salt concentration distributions can be observed in a pavement system and its supporting soil (fig. 6). Because damage due to frost action were soon observed just after construction of this road in 1987, simulations have been made in order to determine its behavior from autumn of 1987 until spring of 1988 (Padilla and Villeneuve, 1989).

In order to realize suitable applications for different conditions of the actual Henri IV boulevard, thermo-physico-chemical properties of the pavement system have been estimated (Laroche, 1988). In this case, air temperature and rainfall are considered to be the climatic diurnal characteristics that would mainly affect the freezing and thawing processes in the pavement. Fig. 7 describes the most prominent features of the Henri IV boulevard structure.

During the winter of 1988, relative frost heave measurements were made at the pavement surface by the *Ministère des Transports du Québec*. Fig. 8 illustrates a comparison of frost heaves calculated by the model and observed in the field. The discrepancy in values for the thawing period are experimentally explained by field observations (Laroche, 1988). In most of the cases, the rate of subsidence of a recent road surfaced with concrete, is smaller than the melting rate of actual ice lenses. During springtime, voids as thick as several centimeters have been observed between the concrete slab and the granular smooth foundations of the pavement structure.

Simulations make us conclude that, under these conditions, the deicing salts (65 t/km) are not significant to the rate of freezing and thawing inside a pavement system. Nevertheless, to overdose such a pavement with an amount of deicing salts as high as 1000 t/km or 2000 t/km can drastically reduce the expected frost heaves and consequently the road damage in this respect (fig. 9).

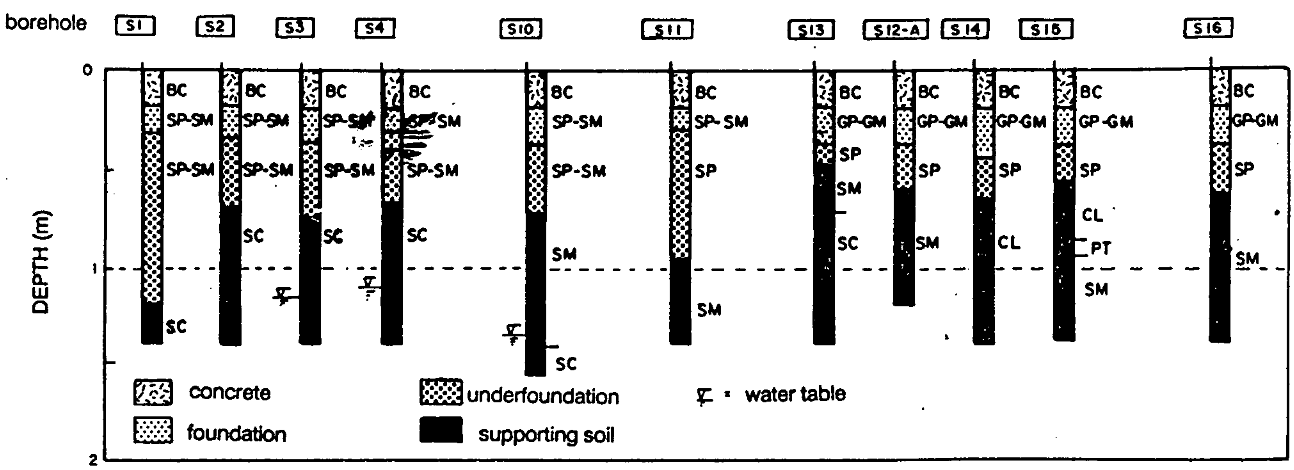

Figure 7. Properties of the Henri IV pavement system (North-right side ; Laroche, 1988).

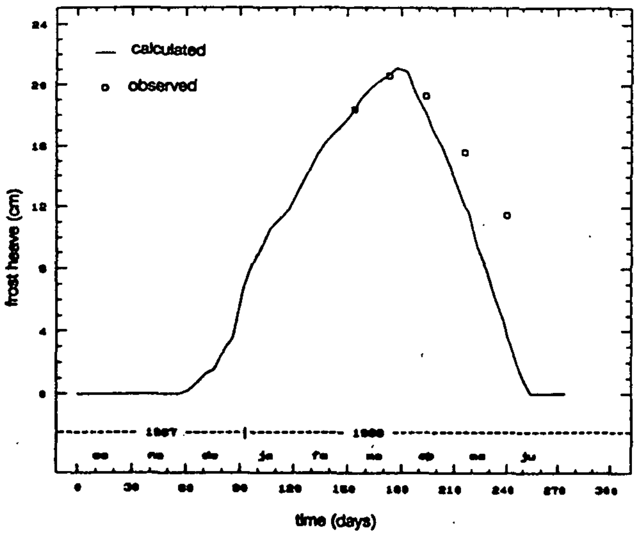

Figure 8. Comparaison of numerical model and observed frost heaves during winter 1987-88 (S3 borehole ; Henri IV Blvd ; Laroche, 1988).

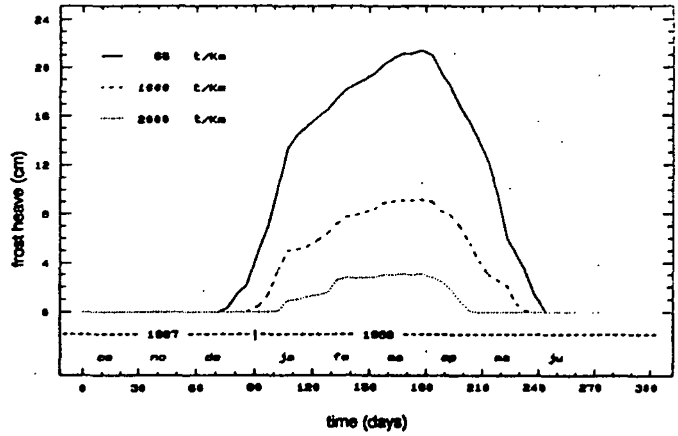

Figure 9. Comparaison of calculated frost heaves for different emlting salt dosages (S3 borehole ; Henri IV Blvd ; Laroche, 1988).

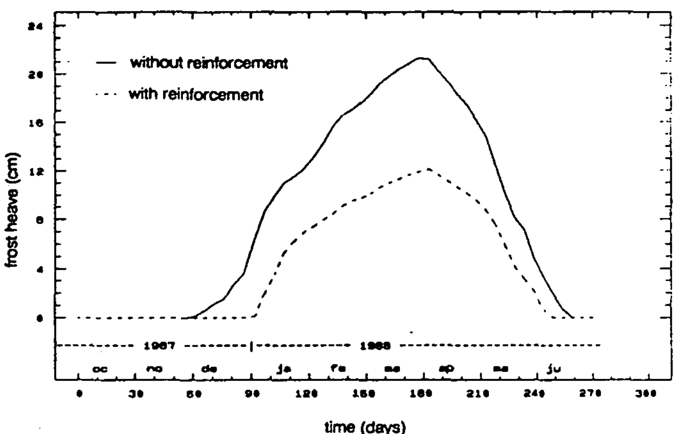

Figure 10. Comparaison of calculated frost heaves for a pavement reinforced or not with a 20 cm thick layer of asphalt (S3 borehole ; Henri IV Blvd ; Laroche, 1988).

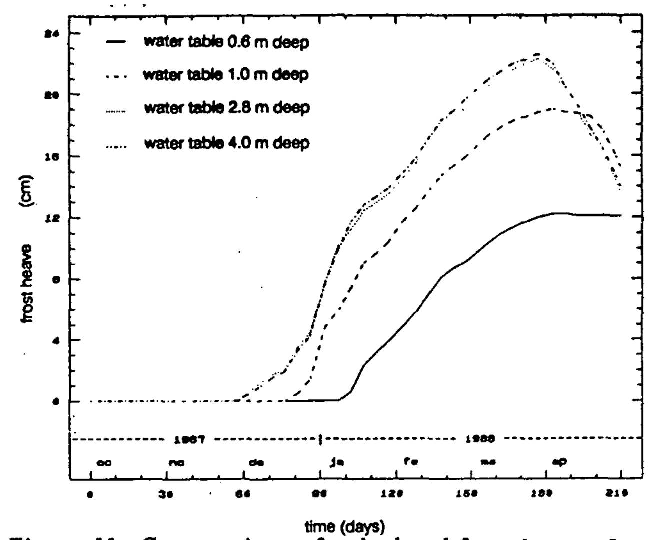

Figure 11. Comparaison of calculated frost heaves for a water table 0.6m, 1.0m, 2.8m and 4.0m deep (S3 borehole ; Henri IV Blvd ; Laroche, 1988).

Reinforcement of a damaged pavement surface increases thermal insulation, diminishes frost heave and consequently extends the original road durability. This phenomenon appears clearly on fig. 10, where the concrete-surfaced road was covered in a 20 cm thick layer of asphalt. In the case of frost-susceptible subsoils, fig. 11 illustrates how a deeper water table can increase frost heave of typical pavement surfaces. That is, stronger capillary forces would prevent more and more the formation of ice into the pore voids of a granular foundation. Poor quality materials would not necessarily heave, but if too much ice is allowed to form in their pores, damages would also be expected because, frequently, the resulting high breaking strains would reduce the resistance of the road foundation structure. Therefore pavement damage due to frost action are not always related to heaving phenomena. Afterwards in this case, water tables deeper than the ones illustrates in fig. 11, would be expected to diminish heaving processes in a frost-susceptible subsoil. Nevertheless, deeper water tables do not reduce significantly the observed heaving rates, specially when these heaving rates are produced by ice lensing in a frost-susceptible supporting soil.

Concluding remarks

The results of the tests described above have established a reasonable degree of confidence in the ability of the model, first, to simulate in a precisely numerical manner, the conditions related to saturated-unsaturated water flow as well as to heat and solute transports in soils. Secondly, the model represents most of the theoretical features concerning ground freezing, i.e. the general behavior of a porous medium due to the different conditions in which ice could be formed inside.

Applications take into account frost heave of actual pavements. Comparisons of calculated heave and experimental frost heave data are better, in general, for sandy soils than for clayey ones. Simulations consider the sensitivity of frost heave to pavement reinforcement, water table depth and deicing salt dosage. In permafrost free zones, actual deicing salt dosages do not significantly affect the observed frost heave of highways. Nevertheless, it can be expected that ground water salinity would represent a significant factor in permafrost related problems.

Because most of the simulations have been performed with estimated parameters, efforts must be made in order to ascertain some soil properties which are specially sensitive to ground freezing. Among these properties we can point out: thermal conductivity, specific heat, moisture retention function, hydraulic conductivity in frozen conditions as well as irreducible degree of saturation (Berg *et al.*, 1980).

Acknowledgements

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