Hydrology of discontinuous permafrost: Effects of permafrost plateau geometry on subsurface drainage

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ABSTRACT

A relationship between the runoff timing and basic geometric parameters for peat plateaus in the discontinuous permafrost zone of the Northwest Territories, Canada was developed. Numerical simulations of idealized plateaus were used to develop the relationship and were compared to the runoff timing of irregular plateaus from the Scotty Creek basin, Northwest Territories.

RÉSUMÉ

Une relation entre le moment où les eaux de ruissellement et les paramètres de base géométrique pour plateaux de tourbe dans la zone de pergélisol discontinu des Territoires du Nord-Ouest, le Canada a été élaboré. Des simulations numériques de plateaux idéalisée ont été utilisés pour développer les relations et ont été comparés à la date du ruissellement des plateaux irréguliers du bassin Scotty Creek, TN-O

1   INTRODUCTION

Runoff generation in the discontinuous permafrost zone of the central Mackenzie River basin is largely dominated by subsurface runoff through the active layer of peat plateaus (Hayashi et al, 2004). Recent studies by Wright et al (2009) have developed a quasi-three dimensional, coupled heat and water transfer model that is able to simulate seasonal frost table thaw and subsequent runoff generation for individual peat plateaus. Although effective in simulating runoff generation for a single plateau, this model is computationally intensive and impractical for modeling the mosaic of peat plateaus that define the discontinuous permafrost zone. Modeling the runoff generation from aggregated plateaus over an entire basin requires a relationship equating the runoff from a single plateau to easily obtained parameters. This paper develops a relationship between the modeled runoff generation from individual peat plateaus and basic plateau geometries using numerical simulation. The relationship is developed by simulating runoff from a number of idealized plateaus, comparing the maximum height, depth to frost table, area, and perimeter to the time a given amount of water would drain into the surrounding bogs and fens. Actual plateau geometries are simulated to validate the relationships. By doing so, a relationship between the area, perimeter, and height of a plateau, and the runoff timing is developed.

The zone of discontinuous permafrost in the wetland dominated northern boreal forest of Canada is a mosaic of peat plateaus underlain with permafrost, and bogs and fens that are seasonally frozen (Robinson, 2000). Figure 1 shows a schematic cross section of a peat plateau in the discontinuous permafrost zone, where the plateaus rise 1 to 2 meters above the surrounding bogs and fens,

and support thriving black spruce forest. The peat plateaus are composed of sphagnum moss and lichens, and are underlain by permafrost. The frost table under these plateaus corresponds closely to the $0^{\circ}C$ isotherm, which evolves annually to maximum depths of 0.5 to 1.0 meters. Hydrologically, the frost table acts as an impermeable boundary, similar to bedrock, so that all subsurface flow is through the thawed active layer (Woo, 1986). The bogs and fens surrounding the peat plateaus are also composed of peat in the upper layer, but are underlain by clay till mineral soil at depths of between 2 and 3 meters, and are only seasonally frozen. Runoff from the peat plateaus is either stored by isolated bogs, or routed through interconnected bogs and channel fens.
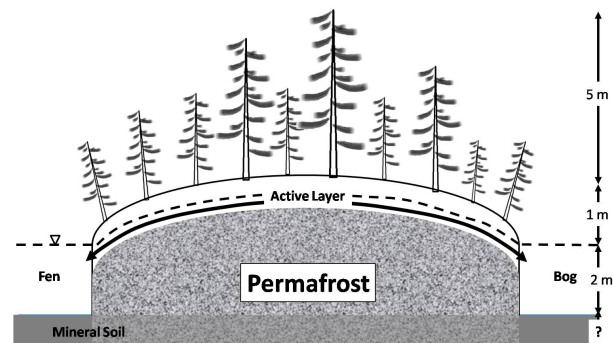

Figure 1. Schematic cross section of peat plateau showing runoff through the saturated active layer along the impermeable permafrost boundary, into the surrounding fens and bogs
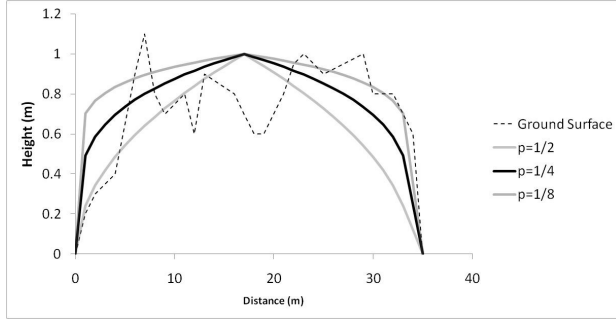
Figure 2. Idealized plateau cross-section using a power function (Equation [4]) and different fitting parameters.

2 THEORY

The Boussinesq equation can be used to describe subsurface flow and saturation along complex hillslopes (Troch et al, 2003). Wright et al (2009) showed that flow through the active layer of peat plateaus can be simulated using the Boussinesq equation, assuming that the permafrost acts as an impermeable boundary and flow is predominantly through the saturated layer. For a radially uniform peat plateau, the equations can be simplified into radial coordinates:

$$n_e \frac{\partial h}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(rT(h) \frac{\partial h}{\partial r} \right) \tag{1}$$

where $h(r,t)$ is the height of the water table, n_e is the drainable porosity, $T(h)$ is the transmissivity of the saturated active layer, r is the radial distance from the plateau center, and t is time. The change in height of the water table, $h(r,t)$, for a complex hillslope can be defined in relation to the saturated thickness, $y(r,t)$, and the height of the frost table, $z(r)$, assuming that the frost table does not evolve within the time frame of a single drainage event (e.g. days):

$$h(r,t) = y(r,t) + z(r) \tag{2}$$

Substituting $h(r,t)$ from equation [1] with equation [2] gives an equation for solving the height of the saturated layer at any given time, which can be used for equating runoff volume:

$$n_e \frac{\partial y}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(rT(y)\left(\frac{\partial y}{\partial r} + \frac{\partial z}{\partial r}\right) \right) \tag{3}$$

From equation [3] it can be seen that the height of the saturated layer is dependent on the slope of the impermeable surface of frozen peat, $z(r)$. Figure 2 shows a transect of a plateau at Scotty Creek, with idealized plateau surfaces defined by a power function:

$$z_s(r) = H\left(1 - \frac{r}{R}\right)^p \tag{4}$$

where $z_s(r)$ is the ground surface elevation with respect to the wetland water level, H is the height of the plateau at a radius of zero, R is the radius of the plateau, and p is a fitting parameter. Figure 2 shows that p=1/4 provides a reasonable fit for the peat plateau. A constant depth to the frost table, D, is assumed in this study, therefore, $z(r) = z_s(r) - D$. Using equation [4] with p=1/4 to represent the topography of the ground surface, $z_s(r)$, and creating the dimensionless form of equation [3], the dimensionless saturated thickness for an idealized permafrost plateau at any given time can be determined by equation [5]:

$$\frac{\partial y_*}{\partial t_*} = \frac{1}{r_*} \frac{\partial}{\partial r_*}\left(r_* T_r \left(\frac{\partial y_*}{\partial r_*} + \frac{H}{4D}(1 - r_*)^{\left(\frac{1}{4}-1\right)} \right) \right) \tag{5}$$

where T_r is the average transmissivity, which can be approximated by $K_0 D$, where K_0 is the depth averaged hydraulic conductivity, and:

$$y_* = \frac{y}{D} \qquad r_* = \frac{r}{R} \qquad t_* = \frac{t}{\tau} \qquad \tau = \frac{n_e R^2}{K_0 D}$$

$$[6a] \qquad\quad [6b] \qquad\quad [6c] \qquad\quad [6d]$$

Equation [5] indicates that, if the average transmissivity is the same for all peat plateaus, runoff generation in dimensionless form is dependent only on a dimensionless variable H/D, referred to as γ. Therefore, a relationship between γ and the runoff generation from an ideal plateau should exist, as the dimensionless radius, r_*, will be consistent for all radially uniform plateaus. Defining runoff generation in terms of timing, the dimensionless time for a given volume of water to runoff of a plateau should only be a function of γ ($t_*(\gamma)$), so that the actual time for a given volume of water to drain, t, can be determined by equation [6c] if the function $t_*(\gamma)$ and τ are known.

The variables for calculating τ have been determined through previous studies in the Scotty Creek research basin. Quinton et al (2008) showed that the hydraulic
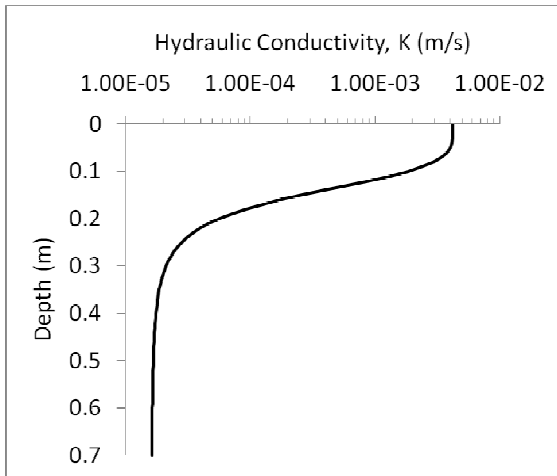
Figure 3. Depth dependant hydraulic conductivity for peat plateaus (after Quinton et al., 2008).

conductivity, K, decreases with depth (see Figure 3), with a lower envelope of 1.6×10^{-5} (m s^{-2}) and an upper envelope of 4.2×10^{-3} (m s^{-2}), transitioning at a depth of 0.15 m. K_o is determined by the harmonic mean of K over the saturated thickness, D. A drainable porosity of $n_e=0.15$ was used as a reasonable value for peat material.

The equivalent radius, R, of non-ideal plateaus is required in equation [6d] to solve for τ. The hydraulic radius (R_{hyd}) is a measure of the area of a polygon in relation to its perimeter, which can be equated to a similar circle:

$$R_{hyd} = \frac{2A}{P} \qquad [7]$$

where A is the area of the plateau, and P is the perimeter. The hydraulic radius is expected to approximate the flow length of an irregular plateau.

The application of the hydraulic radius for determining the equivalent radius of irregular plateaus can be tested by numerically modeling symmetric star shaped plateaus. The shape of a star shaped plateau is represented by a cosine function:

$$R(\theta) = [(1-b)\cos(n\theta) + b]R_o \quad 0.5 < b \le 1 \quad [8]$$

where $R(\theta)$ is the radius of the plateau at a given angle, θ is the rotational angle, R_0 is the maximum radius of the plateau, and b and n are constants defining the shape of the star (see Figure 4). The plateau profile, defined by equation [4], and the irregular plateau radius, defined by equation [8], can be used to develop irregular shaped plateaus with varying hydraulic radii. The appropriateness of using the hydraulic radius as an equivalent plateau radius can be tested by numerically modeling the irregular shaped plateaus, using the simulated results to solve for R in equation [6d], and comparing them to the hydraulic radius.

3   METHODS

3.1   Site Description

The plateaus replicated in numerical modeling are from the Scotty Creek Basin (61°18'N, 121°18'W; 285 m above sea level) within the Mackenzie River Basin of the Northwest Territories, Canada. The plateaus in this area rise between 0.5 to 1.5 meters above the surrounding unfrozen bogs and fens, and are highly irregular in geometry. The maximum depth to the frost table varies between 0.2 meters during the spring thaw and 0.75 meters at the end of the summer (Wright et al. 2009). This corresponds to a minimum γ of 2 and a maximum γ of 10, if the thawed layer is completely saturated. A digital elevation map of the area (Figure 5) was de-trended by subtracting a linear trend from the elevation of each grid cell (1 m by 1 m) to delineate the area, perimeter and height of eight plateaus.

3.2   Runoff Simulation

The relationships for $t(\gamma)$ and R were determined numerically using the Simple Fill and Spill Hydrology (SFASH) model that solves the Boussinesq equation on a two-dimensional model domain, using a finite difference spatial discretization and implicit time scheme with a transmissivity that is dependant on the height of the water table (Wright et al, 2009). The model simulates the flow and storage of water on a peat plateau having an arbitrary geometry and elevation distribution. Grid discretizations of 1 m were used, and the depth to frost table, D, was constant in both time and space for this study.

The plateaus were set as initially saturated, and were
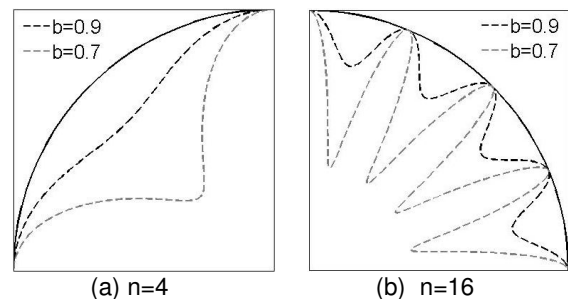

(a) n=4   (b) n=16

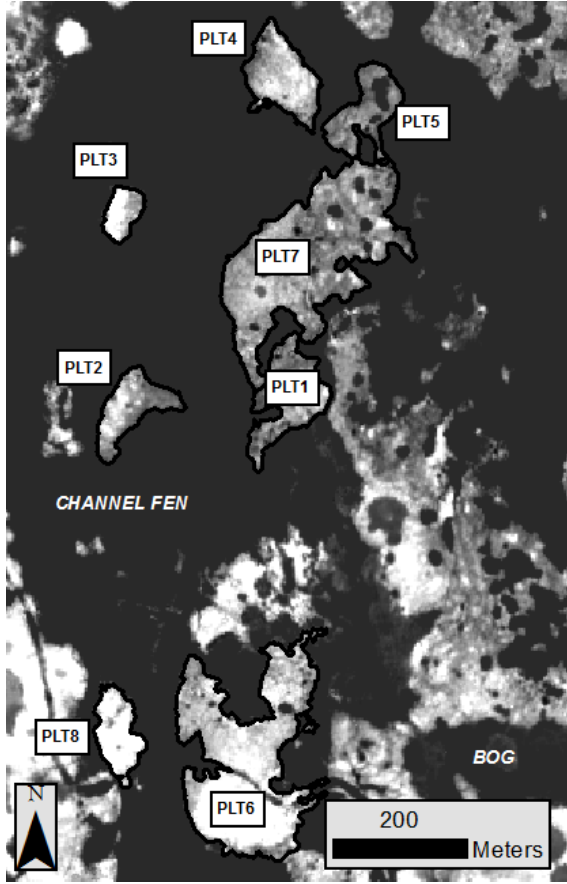Figure 4. Irregular shaped plateaus using equation [8] to determine the radius in polar coordinates.

Figure 6. Detrended digital elevation map for part of the Scotty Creek basin, showing delineated plateaus. Brighter areas have higher elevation (plateaus) than darker areas (wetlands).

modeled for computational efficiency, as the flow was radially symmetric. The ground surface elevation was prescribed by equation [4], and the depth to frost table, D, was determined from H and γ. Simulation results using different combinations of parameters showed that $t_{*80}(\gamma)$ can be represented by a power function for both constant K and depth variable K:

$$t_{*80} = a(\gamma)^b \qquad [9]$$

where the best fit values of the fitting parameters were a = 0.88 and b = -0.96 for simulations with depth-variable K, and a = 0.60 and b = -1.03 for simulations with constant K (see Figure 6). The high correlation coefficient for the best fit line (R^2 = 0.991 for depth-variable and 0.999 for constant conductivity simulations) provides reasonable confidence in Equation [9], although the extents are not well confined. Therefore, the relationship should only be applied within the simulated bounds (γ = 2 to 20).

For a circular plateau having an arbitrary value of γ, the relationship between R and t_{80} is determined by equations [9], [6c], and [6d]. We us this relation to define a hydrologically equivalent radius (R_{eq}) for non-circular plateaus using t_{80} determined by numerical simulation:

$$R_{eq} = \sqrt{\frac{t_{80}K_0 D}{n_e a \gamma^b}} \qquad [10]$$

This is expected to be a better indicator of the hydrological response of plateaus than R_{hyd} (equation [7]), which is purely based on area and perimeter, so that the appropriateness of R_{hyd} can be tested.

To find a relationship between R_{eq} and R_{hyd} for non-circular plateaus, a number of plateaus defined by Equation [8] (see Figure 4) were modeled with γ values between 2 and 20, and b and n values between 0.7 to 0.9 and 4 to 16 respectively. The results for depth-variable K simulations show that R_{eq} is greater than R_{hyd} (see Figure 7) for smaller hydraulic radii (R_{hyd} < 12 m), but equates closely to R_{hyd} at large hydraulic radii. Similar results

drained without recharge until steady state was reached. The time until 80% of the water by volume had drained was taken as t_{80} (or t_{*80} in dimensionless time, see Equation [6c]) and used as an indicator of the hydrological response of the plateau. The dimensionless Boussinesq equation (Equation [5]) indicates that for an idealized circular plateau, t_{*80} should be a function of γ only. The simulations were used to determine the $t_{*80}(\gamma)$ function, and examine if the same hydrological response would occur for plateaus having more complex geometries.

4   RESULTS

4.1   Simulation of Hypothetical Plateaus

To determine the $t_{*80}(\gamma)$ function, circular plateaus with a radius (R_0) of 10 and 20 meters were used with heights of 1 and 2 meters, and γ values of between 2 and 20. Both a constant K and a depth variable K were used during simulation. Only a quarter of each plateau was
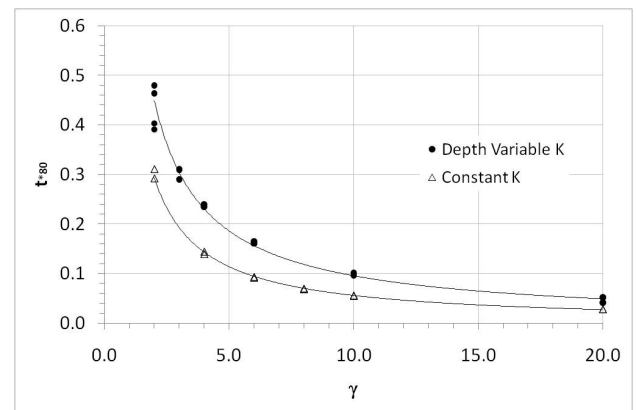

Figure 5. The dimensionless time until 80% of water from a saturated plateau drains in relation to Hi, including the best fit line and correlation coefficient.
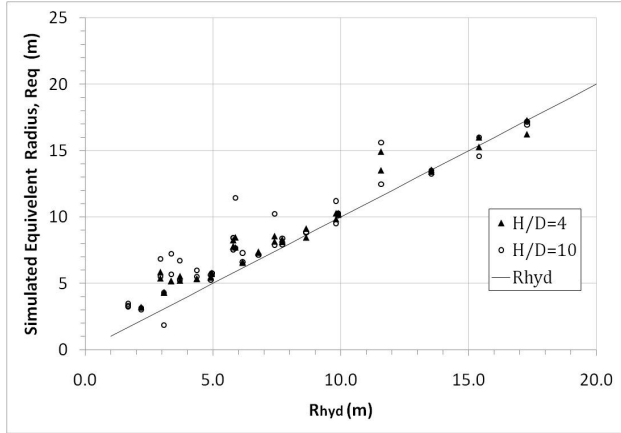
Figure 7. Predicted radius in relation to the hydraulic radius for different γ values, showing a direct relation at large R_{hyd}.

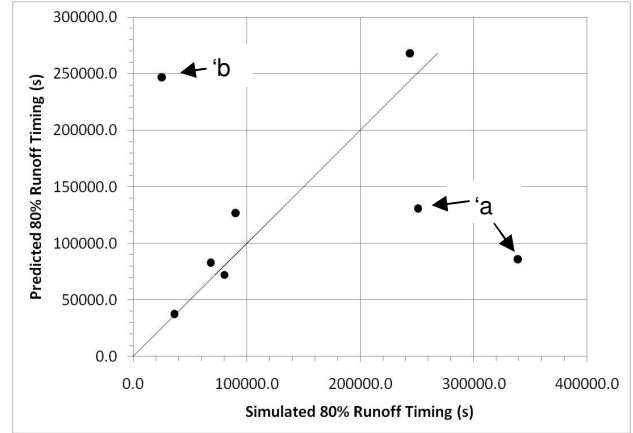

Figure 8. Predicted runoff in relation to the actual simulated runoff for various plateaus from the Scotty Creek basin.

were obtained for constant K simulations (data not shown). This suggests that for plateaus with a relatively large hydraulic radius, R_{eq} (and subsequently R) can be approximated by R_{hyd}.

Substituting R_{hyd} for R in Equation [6d], the runoff timing from peat plateaus can be approximated by equations [6c], [6d], [7], and [9]:

$$t_{80} = \frac{R_{hyd}^2 \eta_e \left(a(\gamma)^b \right)}{K_0 D} \qquad [11]$$

The effectiveness of equation [11] was tested by comparing the numerically simulated t_{80} from actual plateaus from the Scotty Creek basin (see Figure 5) to the t_{80} predicted from equation [11]. For plateaus that were larger than a single model domain, the plateau was divided along flow boundaries, and the bounding elevations raised so that no flow would cross the flow divide. The simulated cumulative storage was then combined for all segments of the plateau to simulate the t_{80} of the aggregated plateau. The results show that the majority of simulated plateaus had a predicted t_{80} similar to the simulated t_{80} (see Figure 8), with three distinct outliers.

5   DISCUSSION

The simulations show that runoff timing from irregular peat plateaus can be approximated using basic geometric properties. From the radial, dimensionless Boussinesq equation (equation [5]) it was expected that there would be a relationship between runoff timing and the plateau height to depth ratio (γ). From numerical simulation it was found that this relation took the form of

a power function for both a constant K_0, and a depth variable K_0 (see Figure 6 and equation [9]).

The consistently faster drainage time associated with the constant K_0 (see Figure 6) is due to a constant K value that was larger than any of the averaged depth variable K_0. For the depth variable K_0, small γ values result in two distinct subpopulations for t_{*80}, with the upper values corresponding to a height of 1 m and the lower values corresponding to a height of 2 m. For small γ values, the difference in D (and subsequently on the harmonic mean of K, see Figure 3) between the plateaus with heights of 1 m and 2 m causes variation in the calculated runoff timing, t_*. This suggests that the use of the harmonic mean for determining K_0 is likely inappropriate, and further work should be done on determining a better K averaging method.

The equivalent plateau radius from equation [10], R_{eq}, converges to the hydraulic radius, R_{hyd}, (see Figure 8) so that the radius of irregular plateaus can be approximated
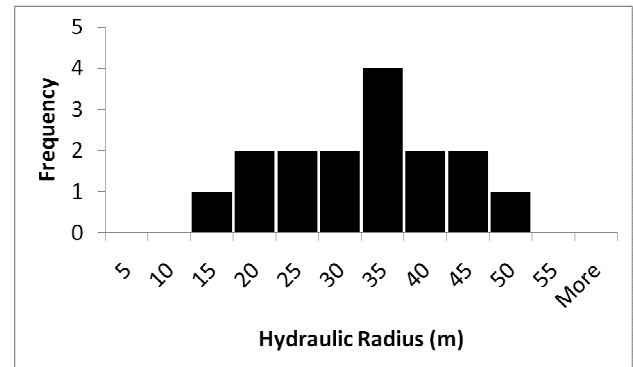

Figure 9. Histogram showing the distribution of hydraulic radii for plateaus within the Scotty Creek basin

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by the hydraulic radius for plateaus with a large R_{hyd} (>12 m). In the Scotty Creek research basin, the majority of plateaus are larger than this threshold, averaging between 15 and 50 meter (see Figure 9), so the use of R_{hyd} in modeling the aggregate of peat plateaus in the Scotty Creek basin is deemed appropriate.

The runoff timing from actual peat plateau simulations was predicted using equation [11] with reasonable success (see Figure 8). The predictions that deviate considerably from the simulated timing had actual plateau flow lengths that deviated from those predicted by the geometrically based hydraulic radius. For the predicted runoff timings that were smaller than the actual timing (see Figure 8, points 'a and Figure 5, PLT1 and PLT4), the flow direction was largely north-westerly rather than in a radial direction. Subsequently, the actual flow length was longer than that predicted by the plateau hydraulic radius, and by equation [11], the actual runoff timing was longer than the predicted timing. For the predicted runoff timings larger than the actual timing (see Figure 8, point 'b and Figure 5, PLT7), internal storage in isolated bogs reduced the area of the plateau draining into the surrounding bogs and fens, so that the area of the plateau generating runoff was significantly reduced. The small area around the edges of the plateau generating runoff into the surrounding bogs and fens have a smaller flow length than that predicted by the hydraulic radius, causing the actual runoff timing to be smaller than that predicted using equation [11]. Future efforts will need to develop a more appropriate method of determining the equivelent plateau radius, accounting for non-radial flow and internal storage.

6   CONCLUSION

This paper developed a relationship between plateau geometries and runoff timing. Using the dimensionless Boussinesq equation, the runoff timing from plateaus is dependent on the height and depth of a plateau. Using this relationship, along with the hydraulic radius to approximate plateau radius, the runoff timing from irregularly shaped plateaus can be calculated. Future efforts developing a better averaging technique for hydraulic conductivity and a more appropriate equivalent radius approximation are required for application of these methods over an entire basin. By combining the equations for runoff timing from individual plateaus developed in this study with a routing algorithm for moving runoff throughout the basin, the hydrological response of an aggregate of peat plateaus in the discontinuous permafrost zone could be determined.

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