Generated Thaw Settlement Profiles Using Autocorrelation Function

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ABSTRACT
Using an autocorrelation function (ACF) to characterize the roughness of a representative ground surface that has experienced thaw settlement is the basis of a new approach to establishing the strain demand on a pipeline subjected to thaw settlement. Simulated settlement profiles are generated by arranging predicted individual borehole settlements from a complete borehole population in a sequence that satisfies the autocorrelation function. The method avoids the difficulty of establishing a design settlement shape and a non-exceedence value for the depth of the design settlement feature. Soil displacements associated with simulated settlement profiles are imposed on the pipe in a pipe structural model to assess settlement strain demand. In this study, an ACF is obtained from published sources to characterize the surface roughness that represents the differential settlement of a specified segment of terrain. By using a numerical interchange scheme, a randomly generated set of thaw settlements is re-arranged to fit the ACF function, without changing any of the original generated settlement values. The effect of the shape of the ACF function, and therefore the simulated settlement profile on predicted pipe strains can be studied using currently available structural analysis programs. Different pipelines respond differently to the generated settlement data.

RÉSUMÉ
Une fonction d'auto-corrélation pour caractériser la rugosité d'une surface qui a subit le dégel est la base d'une nouvelle approche pour établir les stress imposés a un pipeline sous le tassement associé au dégel. Les profils de tassement sont générés en organisant les prédictions de tassement pour chaque trou de forage dans un ordre qui satisfait la fonction d'auto-corrélation. Cette méthode élimine les problèmes associés à la forme que prend le tassement ainsi que les difficultés causés par les valeurs excédents. Les mouvements du sol associés aux profils de tassement estimés sont superposés sur le pipeline dans un model qui représente la tension sur le system. Dans cette étude, la fonction d'auto-corrélation a été obtenue de sources publiées pour caractériser la rugosité d'une surface qui représente le tassement différentiel sur un segment de terrain. En utilisant un arrangement numérique, les valeurs de tassement générées de façon aléatoire, sont réorganisées pour égaler la fonction d'auto-corrélation sans changer le profile de tassement or les valeurs prédites de stress. Les effets de la forme de la fonction d'autocorrection. L'effet du profil et des valeurs simulés peuvent être étudiées par les méthodes d'analyse présentement disponibles. Chaque pipeline répond différemment aux valeurs de tassement générées.

1 INTRODUCTION

Within a soil grouping of similar geological origin in a permafrost region, a family of borehole thaw settlement estimates can be completed for a given pipeline routing. This is normally done by integrating the ice contents or estimated thaw strain values in each borehole with depth, over the anticipated thaw depth interval beneath the pipeline or Right-of-Way in question. Thaw settlement design requires an estimate of "design differential settlement", and needs to establish the shape of the design settlement feature. Typically, the design settlement is taken to be a rationalized non-exceedence value of the estimated thaw settlements in a family of boreholes and the shape of the settlement profile is taken to be a step or a rectangle, e.g. Norman Wells (Nixon et al, 1984) or TAPS pipeline, as shown on Figure 1.

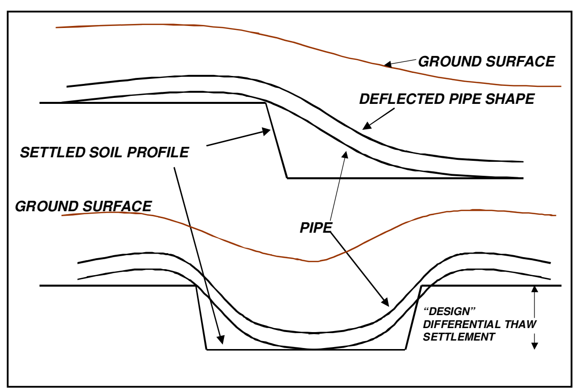

Figure 1. Idealized design thaw settlement profiles

This idealized model of differential thaw settlement can create unnecessary conservatism and also ignores the potential presence of thaw stable sections in generally settling intervals to create settlement strain effects.

Borehole thaw settlement estimates generally seem to follow a log normal distribution with a long ”tail” at the high end of the distribution, rather than a normal distribution. Thaw settlement estimates are often based on widely spaced boreholes, and such groups of settlement estimates do not provide any information on how differential settlement varies from place to place over short distances of concern in pipeline design.

2 BACKGROUND

A method of determining the design differential thaw settlement for northern pipelines in permafrost terrain is required. Various rules of thumb have been adopted in the past such as Differential settlement / Maximum Borehole Settlement = 50% (TAPS) and 75% (Norman Wells). These numbers were based less on scientific analysis, than on the intuitive recognition that sudden differential settlement from zero to the maximum value that could be experienced in a terrain group was not likely to happen. Observations of the differential settlement of foundation columns under large buildings support the use of a factor of about 50%. However, soil arching, structural rigidity and stress interaction between individual footings would tend to generate a more uniform settlement field than may occur beneath a northern pipeline subjected to thaw settlement displacements. Ground ice conditions can vary over relatively short distances, and the surface expression of settlement can be affected by thermal interfaces, where settlement on one side of the thermal interface is essentially zero, and some positive value on the other side of the interface. If differential settlement were set equal to the maximum predicted thaw settlement in any population of settlement estimates, then a high pipeline strain would be predicted. The primary question is how to take a series of borehole thaw settlement estimates such as those shown in Figures 2 and 3, and determine a characteristic differential thaw settlement that can be used in a pipeline structural analysis.

Calculated thaw settlement distributions can be represented quite well by log normal distributions. However, it is not adequate to simply generate random settlement values according to a log normal distribution, and use these directly in a pipeline structural program such as PIPSOL (Nixon, 1995), because adjacent settlement values are un-correlated with each other. That is, some rules must be developed that constrain adjacent settlements from developing too rapidly with distance, otherwise the process becomes unrealistically conservative.

LiDAR data bases show great promise in providing closely spaced input for this kind of thaw settlement analysis. Unfortunately, no suitable LiDAR data are publicly available, hence to illustrate the method, a thaw settlement distribution was obtained from the Mackenzie Valley Pipeline Research data. (Speer et al, 1972).

3 THE AUTO-CORRELATION FUNCTION (ACF)

Autocorrelation is the correlation of a data set with itself, offset by k values. For example, autocorrelation with an offset of 4 would correlate the data set $\{s_0, s_1, s_2, s_3\ s_{n-5}\}$ with $\{s_4, s_5, s_6, s_7 \ldots s_n\}$. The autocorrelation function is the set of autocorrelations with offsets 1, 2, 3, 4 .. limit, where limit is less than or equal to n/2.

The equation for the autocorrelation function (ACF) for a data set, y, with n variables and a mean of $\bar{y}$ is

$$\hat{p}_k = \frac{\frac{1}{n}\sum_{t=k}^{N-(k+1)}(y_t-\bar{y})(y_{t+k}-\bar{y})}{\frac{1}{n}\sum_{t=0}^{N-1}(y_t-\bar{y})^2} \tag{1}$$

This function is related to the auto covariance, with a forward step of *k* elements.

$$ac_k = \frac{1}{n}\sum_{t=k}^{N-(k+1)}(y_t-\bar{y})(y_{t+k}-\bar{y}) \tag{2}$$

The co-variance is normalized by dividing by the variance of the data set, so the ACF is always 1.0 when the offset step (k) is 0 . The function has been used to characterize the roughness of metal surfaces, terrain surfaces from radar imagery, etc.

Palmer (1972) used the function on a small widely spaced thaw settlement data set from the Copper River Basin in Alaska to conjecture that boreholes would have to be drilled closer than 30 m or so, before significant correlation between adjacent borehole thaw settlement estimates might be expected. The available data for adjacent thaw settlement was widely spaced at 30m, and therefore it was difficult to draw conclusions as to how closely boreholes must be positioned before adjacent settlement estimates would correlate. Note that Palmer did not use a normalized ACF, so that his data had units of feet^2 in his publication.It may be possible to characterize the shapes of ACF – distance plots for some sites. Fenton et al (2005) suggests using a Markovian spatial correlation function, an exponential decay of the form

ACF = exp(-2 x / L) (3)

where L is a characteristic length beyond which the data are not significantly correlated. When x = L, the ACF is 0.135 and is only weakly correlated with more widely spaced data points. A distance in the range of 30 to 50 m appears to be the likely characteristic distance.

It remains to be determined how to generate a series of differential thaw settlement values that will “fit” the above ACF-distance functions. Such a string of differential thaw settlement values could be used to determine what characteristic differential thaw settlement value should be used in a pipe structural analysis.

4 MVPL GROUND ICE VARIABILITY SITES

The Mackenzie Valley Pipeline (MVPL) project in the early 1970s studied differential thaw settlement at three sites along their route, Rowley et al, 1972. Settlement predictions for the 2 m to 13 m thaw depth interval were based on a large number of laboratory thaw settlement tests obtained from boreholes spaced about 15 or more meters apart. It appears that almost the maximum differential settlement could be developed over a distance as little as 20 m, for two of the sites, and closer to 50 m for the other.

The grid of calculated thaw settlement values for 15 discrete locations at the Norman Wells site is shown in Figure 2, with the settlement, S, shown in meters.

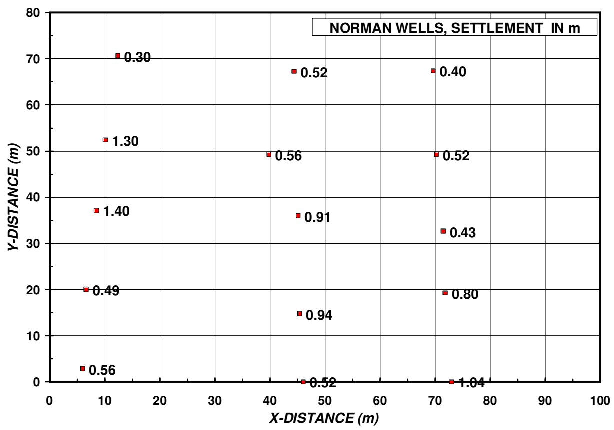

Figure 2. Grid of thaw settlements at MVPL Norman Wells site

A cumulative frequency plot can be prepared for the three sites studied. The higher maximum settlements are noted for the Norman Wells or Landing Lake sites. The Norman Wells site was selected for further analysis, although any of the three sites would provide illustrative thaw settlement values equally well.

A log-normal cumulative frequency distribution has been fitted to the 15 thaw settlement estimates for the Norman Wells location, as shown in Figure 3. The log normal distribution seems to fit groups of thaw settlement estimates better than a normal distribution, which does not capture the high settlement "tail" at the upper end of the distribution. This fact has also been noted by Morgenstern and Collins (1988) when carrying out their study of arching interaction between closely spaced thaw settling locations.

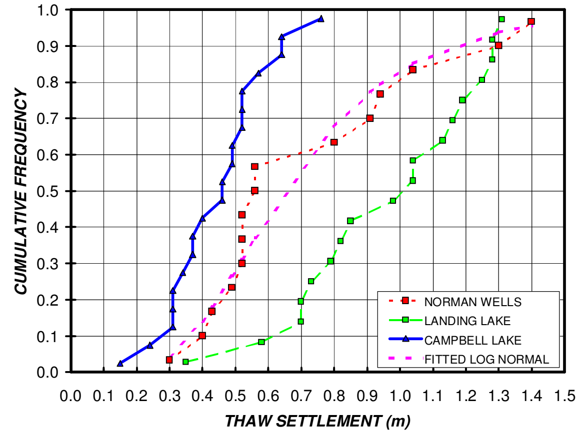

Figure 3. Cumulative frequency of settlement for 3 sites

The same data set can be used to obtain an approximate ACF-distance function. Although settlement estimates (or observations) should ideally be availbale on a much closer spacing of the order of 1 m, the three lines of settlement estimates were used to generate an ACF function for each line. The average spacing of the settlement estimates is about 17 m; the resulting ACF functions have a resolution of roughly this distance. The calculated ACF functions for the three lines of settlement estimates are shown in Figure 4. "Distance" in the plot is the vertical y-distance from Figure 2 (Norman Wells) from the first settlement point in each of the lines.

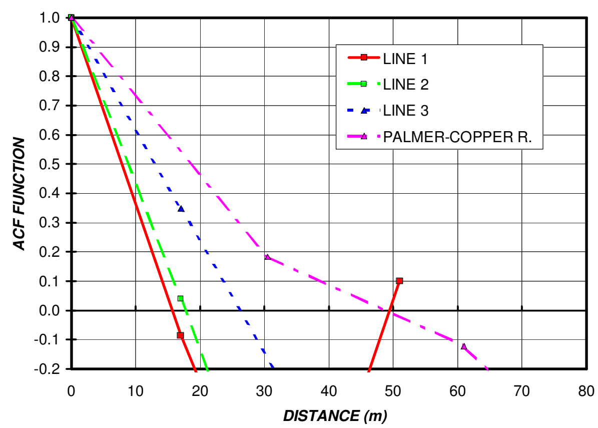

Figure 4. ACF functions for 3 lines of settlement data

The ACF functions intercept zero at some distance around 20 – 30 m, although it is difficult to be precise because the minimum spacing of the settlement estimates is about 17 m. It is of interest to note that the Copper River Basin, Alaska data of Palmer (1972) indicates an ACF function that intercepts zero at a similar spacing of about 30 m.

As LiDAR and possibly other data sources become available and provide closely spaced thaw settlements in thaw settled terrain, it should be possible to obtain more precise ACF-distance functions for use in thaw settlement design. For this study, the ACF-distance function is assumed to be an exponential function of distance, with a characteristic length of 30 m. This is based on the MVPL ground ice variability study sites as described above, together with the tentative results of Palmer (1972) for the Copper River basin in Alaska. The function is plotted later when comparing calculated and target ACF functions.

5 SIMULATED THAW SETTLEMENT PROFILES

The next series of steps in the method generates design differential settlement profiles for use in a pipe structural model.

Using the fitted log normal distribution for the Norman Wells site, the mean and standard deviation for the natural log of settlement are obtained as follows.

- Mean for ln(S) = -0.436, which corresponds to a mean settlement of 0.64 m
- Standard deviation for ln(S) = 0.452

In a two-step procedure that can be completed in a spreadsheet, a series of 1000 random numbers between zero and one are generated and stored. A series of random settlements are obtained that fit the log normal distribution of borehole settlement predictions.

The settlement values are completely unrelated to each other at this point. They do not constitute a realistic representation of differential settlement. The application of the auto-correlation function constrains the manner in which adjacent thaw settlement values can vary in relation to each other.

By using a numerical interchange scheme (Hunter and Kearney, 1983) the randomly generated set of thaw settlements is re-arranged to fit the ACF of the borehole thaw settlements, without changing any of the original generated settlement values. The procedure by which the random settlement values are converted to an auto-correlated sequence that replicates the roughness of a representative settled ground surface is as follows:

1. Calculate the ACF for the randomly generated string of settlements. (On the first iteration, the settlement string is just random “white noise”, and the ACF should be very low at all offsets greater than zero).
2. Compare the ACF-distance function with the target or desired ACF, by calculating the sum of the squares of the residuals at 50 equally spaced points on the ACF function.
3. Randomly select a settlement value in the string and interchange it with another, also randomly selected settlement value in the string.
4. Compare the ACF of the rearranged string with the target ACF. If the result is an improvement in the sum of squares of the residuals, then the interchange is successful, and is retained. If there is no improvement in the comparison of the two ACF, the interchange is considered unsuccessful, and the numbers are returned to their original locations.
5. Repeat steps 3 and 4 a large number of times monitoring the match between the ACF of the rearranged string and the target ACF.
6. When the sum of squares of residuals reaches an acceptably small value, as illustrated in Figure 5, the procedure is terminated and the resulting string of thaw settlement predictions is stored for use in pipe structural analysis.

An example of the results of this procedure is shown in Figure 5. The agreement between the ACF – distance function for the re-arranged settlement profile and the target function is very close.

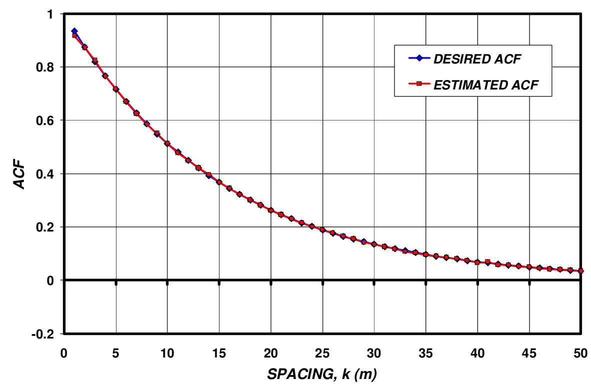

Figure 5. Estimated vs. desired ACF for Norman Wells site

The convergence between calculated and target ACF functions can be monitored as the numerical interchange scheme progresses, as shown in Figure 6.

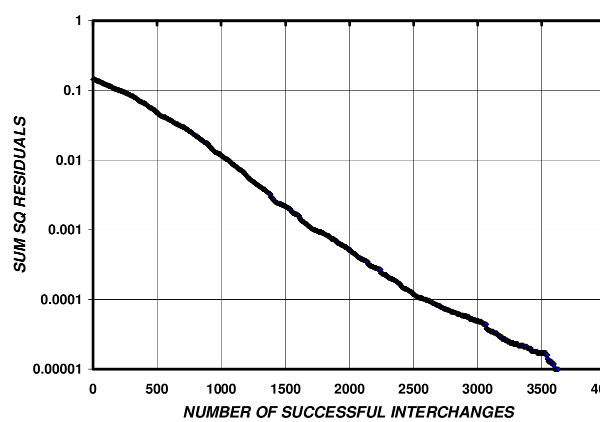

Figure 6. Sum of squares of residuals with number of interchanges

For the example shown, around 3500 successful interchanges were required to reach a tolerance of 10^{-5} , where the tolerance is defined as the sum of the squares of the residual differences between calculated and target ACF functions. It should be noted that the total number of interchanges is much greater (several hundred thousands), as the number of unsuccessful interchanges becomes greater as the simulation progresses. The size of this tolerance required some trial and error, and is sufficiently small that there is no appreciable difference between calculated and target ACF functions.

Figure 7 compares the original randomly generated settlement string and the re-arranged, auto-correlated settlement string. Both of the settlement strings illustrated in Figure 7 follow the log normal distribution of borehole settlements from the MVPL data set used. Figure 8 shows only the re-arranged string, the simulated settlement profile.

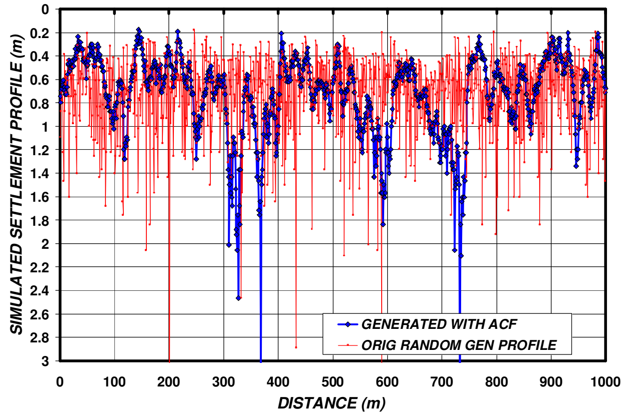

Figure 7. Random and re-arranged thaw settlement profiles

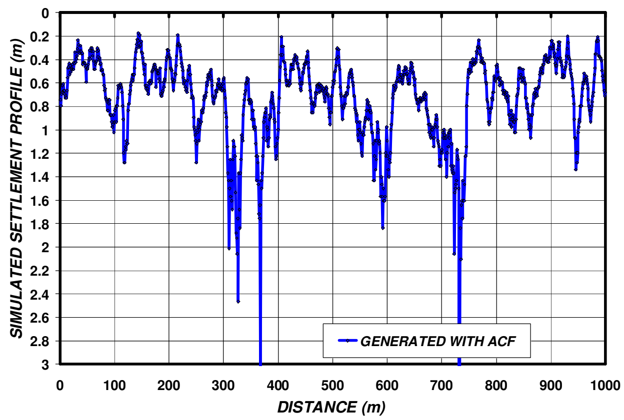

Figure 8. Simulated thaw settlement profile

As stated earlier, the numerical interchange scheme is run a large number of times, until the difference between the calculated and target ACF function is smaller than a specified tolerance.
The simulated settlement profile changes each time the program is run, demonstrating that there is no unique settlement string that satisfies the log normal distribution and the ACF function.

6 PIPE SETTLEMENTS AND STRAINS

The final step in the procedure is to observe the predicted effects on a buried pipe that is subjected to soil loadings exerted by the thaw settled soil profile.

To model the interaction between the pipe and the settled soil, an elastic-plastic soil load displacement function is used in a numerical beam – column pipe structural program. The pipe load is assumed to increase linearly to a peak value at a specified pipe displacement, and the soil is assumed to deform plastically as the pipe displaces continually with no increase in load.

Load-displacement functions that provide adequate definition of the interaction between pipe and soil are calculated using relatively conventional geotechnical methods, and are as follows:

- Vertically downward load 23.92 kN/m at 0.014 m displacement
- Upward (bearing) load 695.60 kN/m at 0.183 m displacement
- Longitudinal (axial) load 16.64 kN/m at 0.018 m displacement.

The pipe properties used in this example are illustrative of a large northern gas pipeline; they do not correspond to any project known to be under consideration at the moment. The pipeline is assumed to be 915 mm (36 inch) in diameter with 15.4 mm wall thickness and a design pressure of 14.5 MPa. The material grade is X-80, which defines the stress-strain properties of the pipe material. The pipe was assumed to be initially horizontal, resting on level terrain. The subsoils are assumed to settle to the simulated settlment profile. The simulation was carried out using the Nixon Geotech Ltd pipe structural analysis program PIPSOL (Nixon, 1994). The predicted soil and pipe settlements for the 1000 m long simulated settlement profile are shown in Figure 9.

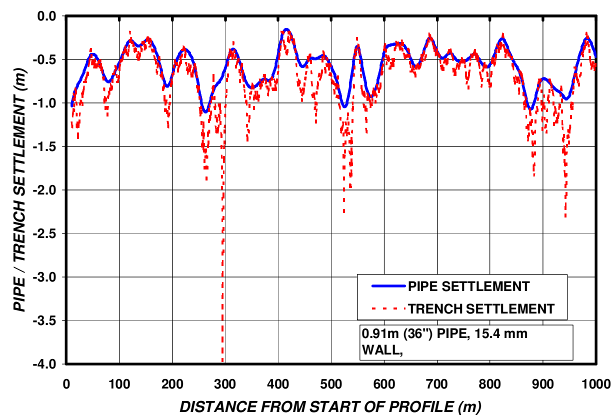

Figure 9. Predicted pipe and soil displacements

As shown, the pipe tends to span or bridge the settlement features, and only responds to settlement areas of the order of 50 m or more. This is very much a function of the stiffness of the pipe section considered. Smaller or thinner pipes will be more compliant, and will tend to follow the simulated settlement profile more closely. Stiffer pipe sections with larger diameter or heavier wall thickness will tend to span settlement features to a greater extent.

A closer view of one of the larger settled areas is given in Figure 10.

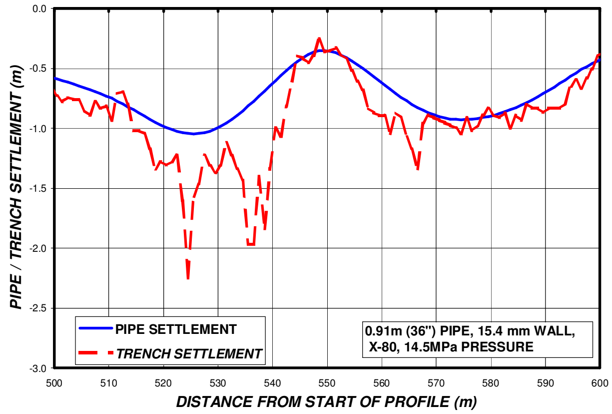

Figure 10. Close-up of pipe and soil settlements

The pipe indents the soil at some of the high points where adjacent soils have settled, and the high points provide support for the pipeline.

The predicted pipe strains in the top and base fibre of the pipe are shown plotted in Figure 11. In this case, the peak strain is about 0.5% at one location, and is more commonly around 0.2%.

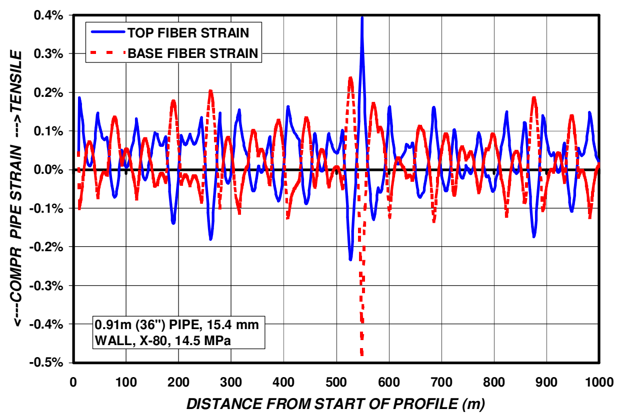

Figure 11. Predicted pipe strains with distance

Again, a close up view of the predicted pipe strains is provided in Figure 12, for the same high settlement area of the thaw settlement profile.

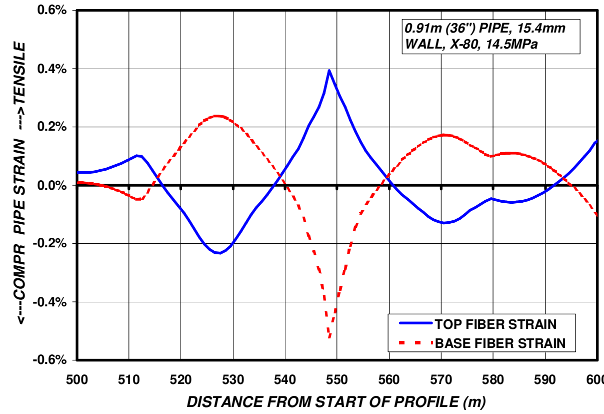

Figure 12. Close-up pipe strains near high settlement area

Figure 12 illustrates that high pipe strains are not just associated with areas of high settlement. High strains can equally well occur at stable locations within generally settling areas. That is, a "hard spot" within settling terrain can result in pipe strains of similar magnitude to the more commonly considered settlement "bowl" surrounded by thaw stable soils.

The relative pipe strains produced by a settling bowl, as compared to a hard spot, likely depends to some extent on the soil load displacement functions selected for the analysis. These in turn depend on soil type, pipe burial depth and water table elevation.

7 SUMMARY AND CONCLUSIONS

- A new approach has been developed for obtaining thaw settlement profiles for northern pipeline design.
- The proposed approach utilizes an autocorrelation function to characterize the roughness of a representative surface that has experienced thaw settlement.
- Simulated settlement profiles are generated by re-arranging randomly generated settlements based on a complete settlement population from a similar geological origin to satisfy the autocorrelation function.
- The method avoids the difficulty of establishing a design settlement shape and a non-exceedence value for the depth of the design settlement feature.
- The simulated settlement can be imposed on the pipe in a pipe structural model to assess settlement strain demand.
- Resulting pipe strains depend largely on the stiffness of the pipe section, derived from its diameter and wall thickness.
- The maximum pipe strain in a selected profile may not be associated with a traditional settlement "bowl", but rather a central elevated "hard spot" surrounded by two zones of settling soil.

8 ACKNOWLEDGEMENT

John Greenslade of JGG Inc in Calgary provided extensive encouragement for the development and application of this design approach.

9 REFERENCES

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